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Economic and Economic-Statistical Designs of the Side Sensitive Synthetic Coefficient of Variation Chart

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Abstract: Control charts are useful tools to monitor signs of assignable cause(s) that results in poor quality products and services, especially in engineering applications. By convention, control charts detect shifts in the mean (μ) and standard deviation (σ). However, certain processes do not have a consistent μ and σ. For such processes, conventional charts will result in dubious conclusions. This motivated the development of charts monitoring the relationship between σ and μ instead, through the coefficient of variation (γ). The side sensitive synthetic chart was recently proposed to monitor γ. However, it is designed based on statistical considerations, i.e., by minimizing the expected number of samples required to detect a specific shift, while concurrently satisfying constraints in the false alarms. The weakness in this design is that it ignores the cost of the chart, which is important for most practical applications. Thus, we will propose economic and economic-statistical designs for the side sensitive synthetic γ chart, in which the former ignores statistical performance, while the latter incorporates statistical constraints. A Scicoslab program is developed to implement these two designs. The impact of various cost and process parameters are also studied.

Keywords: *Coefficient of variation, control charts, economic-statistical design, side sensitive, synthetic chart*

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I. INTRODUCTION

Control charts are designed to detect special cause(s) that is the main cause of unsatisfactory process output, by utilizing sample information to see whether the sample provides evidence that there is a change in the process, typically by monitoring the mean (μ), standard deviation (σ) or range (R). However, in recent years, control charts have been extended to various fields where these parameters are not consistent, even during the in-control phase. Some examples are shown in [1]. This motivated [2] to propose a chart to monitor the coefficient of variation (γ(), where $\gamma = \frac{\sigma}{\mu}$. However, the γ chart proposed by [2] is insensitive to small and moderate shifts. Numerous improvements are then made on the γ chart. [1] proposed an Exponentially Weighted Moving Average (EWMA) chart, followed by [3] who proposed a synthetic chart; [4] who proposed a Variable Sampling Interval (VSI) chart; and [5], who proposed a Side Sensitive Group Runs chart (SSGR). More recent studies on γ charts include [6] who proposed a Variable Sample Size (VSS) EWMA chart; [7] who proposed a new adaptive EWMA chart; [8] who proposed a Variable Sample Size and Sampling Interval (VSSI) chart; and [9] who proposed a side sensitive synthetic chart.

For control charts, a shift in the statistical parameters mentioned in the preceding paragraph is considered an out-of-control condition, and such a shift should preferably be detected quickly by the chart. The charting param-

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eters significantly influences the performance. By convention, these parameters are determined to optimize the chart's statistical performance. The Average Run Length (ARL) is the conventional performance metric, which consist of the in-control ARL (ARL_0) and out-of-control ARL (ARL_1). The ARL_0 measures the expected number of samples until a false alarm occurs, while the ARL_1 measures the expected number of samples until changes in the statistical parameter is detected. A good chart should have a large ARL_0 and small ARL_1. When the chart is designed based on statistical considerations, the charting parameters are selected to minimize the ARL_1, while satisfying constraints in the ARL_0. A commonly used ARL_0 is 370.4.

The statistical design in the preceding paragraph is designed in such a way that it optimizes the detection effectiveness. Statistical designs ignore the cost, which is a concern especially in a production environment when the cost of sampling, testing, failure in detecting shifts etc can be prohibitively high. [10] was the pioneer who first proposed economic models for control charts, which was later generalized by [11] and simplified by [12]. Subsequently, [13] introduced statistical constraints in the economic model. Economic models for the synthetic chart was first introduced by [14] and [15], and is subsequently extended to the multivariate case by [16]. However, these studies are based on designs for the process mean. This motivated [17] to propose economic models for the synthetic γ chart. Besides economic models for synthetic-type charts, economic models for other charts can also be found in the literature, some of the recent ones are [18, 19, 20].

Although the economic designs for synthetic γ charts were proposed by [17], economic models for the side sensitive synthetic γ chart is not available. Thus, this research will propose economic and economic-statistical designs for this chart, which was recently proposed by [9] to improve the synthetic γ chart. Note that the synthetic chart was first proposed by[21] to monitor the process mean. Since then, numerous extensions and improvements are available, for example by [22, 23, 24, 25, 26, 27, 28, 29, 30].

The synthetic chart operates by looking at the proximity between two samples that fall outside the control limits, while an additional feature is included for the side sensitive synthetic chart by [9], where the samples must belong to the same side of the centreline. [9] designed the chart based on statistical considerations, and economic models are not available. This research will fill this gap. Two designs are considered. In the economic design, the charting parameters are selected to minimize cost; while in the economic-statistical design, where besides minimizing cost, statistical constraints need to be satisfied.

The next section describes the economic models, followed by section 3 which shows the results for various numerical examples. Lastly, concluding remarks are given in section 4.

A. *Economic and Economic-statistical Designs of The Side Sensitive Synthetic γ Chart*

The economic models are shown in this section. [9] gives a detailed description of the side sensitive synthetic γ chart.

[11] generalized model is adopted. The process is assumed to be in-control initially, where $\frac{\sigma}{\mu} = \gamma_o$. The time until the occurrence of an assignable cause is exponentially distributed with parameter λ.

Fig. 1 shows a quality control cycle, which is made up of successive in-control and out-of-control periods.

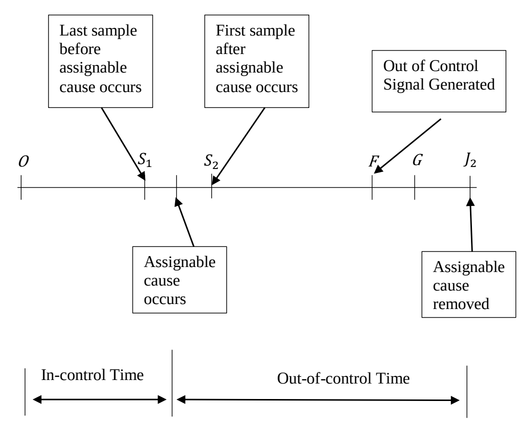

Fig. 1. Quality control cycle

In Fig. 1, OJ_2 shows the full length of a cycle, with OJ_1 denoting the in-control period and $J-1J_2$ denoting the out-of-control period. The total cost incurred in this cycle includes the cost of non-conformities, false alarms, sampling, and to bring the process back to an in-control condition. The ratio of the expected cost over the expected cycle time gives us the estimated cost per unit time (C), as follows:

$$C = \frac{\frac{C_0}{\lambda} + C_1 B + \frac{b+cn}{h}\left(\frac{1}{\lambda} + B\right) + \frac{sY}{ARL_0} + W}{\frac{1}{\lambda} + \frac{(1-\psi_1)sT_0}{ARL_0} + EH} \tag{1}$$

with

$$B = (ARI_1 - 0.5)\,h + F$$
$$F = ne + \psi_1 T_1 + \psi_2 T_2$$
$$EH = (ARI_1 - 0.5)\,h + G$$
$$G = ne + T_1 + T_2$$
$$s = \left(\frac{1}{\lambda h}\right) - \left(\frac{1}{2}\right)$$

The parameters used in the cost function are defined as follows:

b = fixed costs per sample,
c = costs per sample unit,
C = expected costs per hour,
C_0 = expected in-control quality cost,
C_1 = expected out-of-control quality cost,
e = expected time to sample and interpret one unit,
h = sampling interval,
n = sample size,
s = expected number of samples that will be taken before an assignable cause occurs,
T_0 = expected search time for a false alarm,
T_1 = expected time to find the assignable cause,
T_2 = expected time to repair the process,
W = cost of eliminating an assignable cause,
Y = cost of a false alarm,
ψ_1 = 0 if production is stopped during search,
= 1 if the production continues during search,
ψ_2 = 0 if production is stopped during repair,
= 1 if the production continues during repair,
λ = process failure rate.

For the economic design, the optimal charting parameters (n^*, l^*, k^*, h^*) are the combinations that minimizes Eq. 1, whereas for the economic-statistical design, they are chosen among the combination that satisfies ARL_0 and ARL_1 constraints. The optimal h are obtained by solving $\frac{\partial C}{\partial h} = 0$, which will enable us to obtain h as

$$h = \frac{-r_2 + \sqrt{r_2^2 - 4r_1 r_3}}{2r_1} \tag{2}$$

with:

$$r_1 = \frac{(ARL_1 - 0.5)\left\{\lambda\left(Y + C_1 T_0(-1+\psi_1)\right) - 2ARL_0\left[C_0 + \lambda\left((ARL_1-0.5)\,b + (ARL_1-0.5)\,cn + W\right) + C_1(-1+F\lambda - G\lambda\right]\right\}}{2r_1},$$
$$r_2 = \frac{2\,(ARL_1-0.5)\left[Y + C_1T_0(-1+\psi_1) + ARL_0(b+cn)(1+F\lambda\right]}{\lambda ARL_0},$$
$$r_3 = -\frac{1}{2\lambda^2 ARL_0}\left[\begin{array}{c} 2Y + 2C_0T_0(-1+\psi_1) - bT_0\lambda - 2\,(ARL_1-0.5)\,bT_0\lambda - 2C_1FT_0\lambda - cnT_0\lambda \\ -2\,(ARL_1-0.5)\,cnT_0\lambda - 2T_0W\lambda + 2GY\lambda + bT_0\psi_1\lambda + 2\,(ARL_1-0.5)\,bT_0\psi_1\lambda \\ -cFnT_0\lambda^2 + bFT_0\psi_1\lambda^2 + cFnT_0\psi_1\lambda^2 + 2ARL_0(b+cn)(1+F\lambda)(1+G\lambda) \end{array}\right]$$

[14] shows the derivations for Equation 2.

This paper considers L between 1 to 30 and n between 2 and 30. The maximum value for L is set as 30 because past studies show that the minimum cost is usually achieved before $L = 30$ [14, 17]. We also set the maximum n as 30 since practitioners rarely choose sample sizes greater than 30.

The following are the steps to obtain (n^*, L^*, k^*, h^*) :

1: Initialize (L, n) = (1, 2)
2: Obtain k that minimizes C
3: Calculate ARL_0 and ARL_1
4: Calculate h from Eq 2
5: Calculate C from Eq 1
6: Increase L by 1 without changing n
7: Repeat 2-6 until $L = 30$.
8: Reinitialize L to 1, then increase n by 1.
9: Repeat 2-8 until $n = 30$.

From all the combination of (n, L, k, h) from Steps 1 to 9, the combination which gives the lowest C is the optimal combination for the economic design. However, for economic-statistical design, the optimal combination also has to satisfy $ARL_0 \geq 250$ and $ARL_1 \leq 1$. A Scicoslab program is developed to implement the designs.

II. NUMERICAL EXAMPLES

This section shows the results based on numerical examples, with input parameters as shown in Table 1. The same examples are used by [17]. The input parameters in Cases 2, 5, 10, 13, 16, 19, 21, 25, 28, 31, 34, 37, and 39 are all the same.

TABLE 1
NUMERICAL EXAMPLES

Case	λ	τ	C_0 ($)	C_1 ($)	Y($)	W($)	b($)	c($)	e	T_0	T_1	T_2	ψ_1	ψ_2
1	0.01	1.50	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
2	0.02	1.50	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
3	0.04	1.50	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
4	0.02	1.25	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
5	0.02	1.50	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
6	0.02	1.75	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
7	0.02	2.00	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
8	0.02	2.50	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
9	0.02	1.50	57.12	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
10	0.02	1.50	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
11	0.02	1.50	228.48	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
12	0.02	1.50	114.24	474.6	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
13	0.02	1.50	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
14	0.02	1.50	114.24	1898.4	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
15	0.02	1.50	114.24	949.2	488.7	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
16	0.02	1.50	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
17	0.02	1.50	114.24	949.2	1954.8	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
18	0.02	1.50	114.24	949.2	977.4	488.7	0	4.22	0.083	0.083	0.083	0.75	1	0
19	0.02	1.50	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
20	0.02	1.50	114.24	949.2	977.4	1954.8	0	4.22	0.083	0.083	0.083	0.75	1	0
21	0.02	1.50	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
22	0.02	1.50	114.24	949.2	977.4	977.4	5	4.22	0.083	0.083	0.083	0.75	1	0
23	0.02	1.50	114.24	949.2	977.4	977.4	10	4.22	0.083	0.083	0.083	0.75	1	0
24	0.02	1.50	114.24	949.2	977.4	977.4	0	2.11	0.083	0.083	0.083	0.75	1	0
25	0.02	1.50	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
26	0.02	1.50	114.24	949.2	977.4	977.4	0	8.44	0.083	0.083	0.083	0.75	1	0
27	0.02	1.50	114.24	949.2	977.4	977.4	0	4.22	0.042	0.083	0.083	0.75	1	0
28	0.02	1.50	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
29	0.02	1.50	114.24	949.2	977.4	977.4	0	4.22	0.166	0.083	0.083	0.75	1	0
30	0.02	1.50	114.24	949.2	977.4	977.4	0	4.22	0.083	0.042	0.083	0.75	1	0
31	0.02	1.50	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
32	0.02	1.50	114.24	949.2	977.4	977.4	0	4.22	0.083	0.166	0.083	0.75	1	0
33	0.02	1.50	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.042	0.75	1	0
34	0.02	1.50	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
35	0.02	1.50	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.166	0.75	1	0
36	0.02	1.50	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.375	1	0
37	0.02	1.50	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
38	0.02	1.50	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	1.5	1	0
39	0.02	1.50	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
40	0.02	1.50	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	0	0
41	0.02	1.50	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	0	1
42	0.02	1.50	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	1

Using the methods described in Section 3, the result of each case is obtained and shown in Table 2.

TABLE 2
(N^*, L^*, K^*, H^*) AND THE CORRESPONDING (C^*, ARL_0, ARL_1) FOR BOTH DESIGNS OF THE SIDE SENSITIVE SYNTHETIC γ CHART

	Economic Design							Economic-statistical Design							
Case	n^*	L^*	k^*	h^*	C^*($)	ARL_0	ARL_1	n^*	L^*	k^*	h^*	C^*($)	ARL_0	ARL_1	% Increase
1	7	5	1.86	1.60	176.37	112.17	2.44	7	6	2.10	1.37	177.52	258.43	2.83	0.66
2	6	6	1.92	1.00	207.97	121.61	2.79	6	6	2.10	0.86	209.55	251.97	3.23	0.76
3	5	7	1.97	0.62	256.26	127.46	3.27	5	8	2.18	0.53	258.44	252.49	3.81	0.85
4	5	12	1.9	0.64	253.35	63.88	6.87	7	13	2.26	0.54	268.12	252.88	8.90	5.83
5	6	6	1.92	1.00	207.97	121.61	2.79	6	6	2.10	0.86	209.55	251.97	3.23	0.76
6	5	5	2.01	1.02	190.33	200.44	2.15	5	5	2.07	0.98	190.43	253.34	2.22	0.05
7	4	5	2.13	0.94	181.40	289.30	2.01	4	5	2.13	0.94	181.40	289.30	2.01	0.00
8	4	4	2.22	1.10	172.35	496.16	1.50	4	4	2.22	1.10	172.35	496.16	1.50	0.00
9	6	6	1.92	0.96	154.66	121.61	2.79	5	8	2.18	0.69	156.29	252.49	3.81	1.06
10	6	6	1.92	1.00	207.97	121.61	2.79	6	6	2.10	0.86	209.55	251.97	3.23	0.76
11	6	6	1.92	1.08	314.34	121.61	2.79	6	6	2.10	0.92	315.82	251.97	3.23	0.47
12	8	5	1.84	2.10	178.24	103.50	2.18	7	6	2.10	1.57	179.38	258.43	2.83	0.64
13	6	6	1.92	1.00	207.97	121.61	2.79	6	6	2.10	0.86	209.55	251.97	3.23	0.76
14	5	7	1.98	0.56	250.36	132.43	3.29	5	8	2.18	0.48	252.51	252.49	3.81	0.86
15	4	7	1.83	0.73	202.64	74.11	3.47	5	8	2.18	0.69	207.04	252.49	3.81	2.17
16	6	6	1.92	1.00	207.97	121.61	2.79	6	6	2.10	0.86	209.55	251.97	3.23	0.76
17	7	6	2.06	1.09	213.20	218.44	2.74	7	6	2.10	1.05	213.30	258.43	2.83	0.05
18	6	6	1.92	0.99	198.85	121.61	2.79	6	6	2.10	0.85	200.45	251.97	3.23	0.80
19	6	6	1.92	1.00	207.97	121.61	2.79	6	6	2.10	0.86	209.55	251.97	3.23	0.76
20	6	6	1.92	1.01	226.19	121.61	2.79	6	6	2.10	0.87	227.76	251.97	3.23	0.69
21	6	6	1.92	1.00	207.97	121.61	2.79	6	6	2.10	0.86	209.55	251.97	3.23	0.76
22	8	5	1.81	1.44	212.20	91.66	2.13	8	6	2.09	1.21	214.56	251.56	2.51	1.11
23	9	4	1.72	1.72	215.40	77.25	1.91	9	5	2.05	1.42	218.34	252.09	2.28	1.37
24	6	7	2.11	0.65	192.72	228.30	3.16	6	7	2.14	0.63	192.75	257.80	3.23	0.01
25	6	6	1.92	1.00	207.97	121.61	2.79	6	6	2.10	0.86	209.55	251.97	3.23	0.76
26	5	6	1.77	1.29	227.82	67.59	2.87	5	8	2.18	0.99	233.99	252.49	3.81	2.71
27	8	5	1.85	1.31	203.92	107.80	2.19	8	6	2.09	1.13	205.63	251.56	2.51	0.84
28	6	6	1.92	1.00	207.97	121.61	2.79	6	6	2.10	0.86	209.55	251.97	3.23	0.76
29	4	8	2.02	0.70	213.73	128.56	3.97	4	9	2.24	0.58	215.35	250.88	4.75	0.75
30	6	6	1.92	1.00	207.97	121.61	2.79	6	6	2.10	0.86	209.55	251.97	3.23	0.76
31	6	6	1.92	1.00	207.97	121.61	2.79	6	6	2.10	0.86	209.55	251.97	3.23	0.76
32	6	6	1.92	1.00	207.97	121.61	2.79	6	6	2.10	0.86	209.55	251.97	3.23	0.76
33	6	6	1.92	0.99	207.38	121.61	2.79	6	6	2,1	0.85	208.97	251.97	3.23	0.76
34	6	6	1.92	1.00	207.97	121.61	2.79	6	6	2.10	0.86	209.55	251.97	3.23	0.76
35	6	6	1.92	1.00	209.15	121.61	2.79	6	6	2.10	0.86	210.74	251.97	3.23	0.76
36	6	6	1.92	1.00	209.43	121.61	2.79	6	6	2.10	0.86	211.03	251.97	3.23	0.76
37	6	6	1.92	1.00	207.97	121.61	2.79	6	6	2.10	0.86	209.55	251.97	3.23	0.76
38	6	6	1.92	0.99	205.10	121.61	2.79	6	6	2.10	0.85	206.67	251.97	3.23	0.76
39	6	6	1.92	1.00	207.97	121.61	2.79	6	6	2.10	0.86	209.55	251.97	3.23	0.76
40	6	6	1.91	1.00	206.33	116.84	2.77	6	6	2.10	0.85	207.97	251.97	3.23	0.79
41	6	6	1.91	1.02	219.95	116.84	2.77	6	6	2.10	0.87	221.64	251.97	3.23	0.77
42	6	6	1.91	1.02	221.60	116.84	2.77	6	6	2.10	0.87	223.23	251.97	3.23	0.74

From Table 2, we can study the impact of the 14 input parameters. From cases 1, 2 and 3, we can study the impact of the process failure rate (λ). It shows that when increases, the expected cost increases. Also, the values of L^*, k^*, ARL_0, ARL_1 increase when λ increases. With a larger λ, more sampling is required as indicated by the reduction in h^*. Increase in influences the optimal L^*, k^*, h^*, C^*, ARL_0 and ARL_1 as shown from cases 4 to

8. The bigger shift results in larger k^*, h^* and ARL_0 and smaller L^*, C^*, and ARL_0. When is large, detecting the shift requires less time. When is large, C^* will be smaller because C_1 is substantially higher than C_0 . In addition, larger shifts in the process are easier to detect. Therefore when is large, it is possible to use less stringent control limits. Lastly, larger ARL_0 indicate fewer false alarms due to less stringent control limits. Cases 9 to 14 shows that when C_0 or C_1 increases, C^* increases as well. A larger C_0 shows larger h^*, but it is the converse for C_1. It can also be observed that the quality cost when the process is in control is more significant than that for the out of control process.

Table 2 shows that when Y increases, it is associated with larger optimal n^*, k^*, h^* and C. A larger k^* leads to a slower detection of the shift. To overcome this, a larger n^* is used. Since an increase in n^* increases the sampling cost, a larger h^* is needed. From cases 18 to 20, W has no obvious effect on the optimal values. Next, from cases 21 to 23, a larger b is associated with larger n^* and h^*, and smaller L^* and k^*. Additionally, as b increase, both ARL_0 and ARL_1 decrease. Thus, the optimal charting parameters and ARLs are significantly affected by the value of b, while the optimal cost is only slightly affected. Cases 24 to 26 show the optimum h^* and C^* increase as c increases. When c increases, the optimum k^* and n^* decrease. Cases 27 to 29 shows that larger e is associated with larger L^*, k^*, cost and ARLs, and smaller n^* and k^*. Cases 30 to 38 show the T_0 has no impact on the optimal values. Similarly, the T_1 only has a minor impact on the optimum cost. This shows that when a longer T_1 is needed, the optimum cost only increases slightly. On the other hand, when T_2 increases, the optimum cost decreases slightly. The last column of Table 2 shows only a marginal increase when statistical constraints was added, but a large improvement in ARL_0 is shown.

III. CONCLUSION

This paper develops economic models for the side sensitive synthetic chart. Numerical result shows that higher values of C_0, C_1, Y, W, b, c, e, and T_1 result in higher cost, while higher values of τ and T_2 result in lower costs. However, larger values of T_0 show negligible effect on the optimal costs. This paper also shows that statistical contraints results in a marginal increase in cost but significantly improves the statistical performance. One of the limitations of this paper is it assumes the exact value of the shift size is known, which may not be possible in certain scenarios. Future research can develop economic models for unknown shift sizes. Furthermore, this paper studies processes with a single quality characteristics with a single assignable cause. Future studies can look into developing economic models for multivariate processes and/or processes with multiple assignable causes.

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