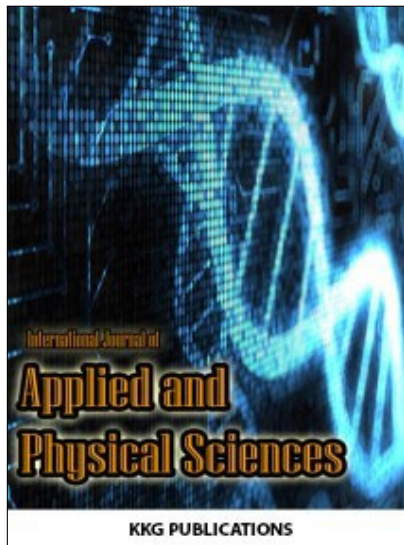


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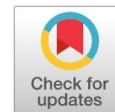
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# GREY FORECAST MODELS OF MANPOWER DEMAND FOR PILOTS IN TAIWAN

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**Abstract.** Airmen are one of the most important resources in the air transport industry. They are qualified and professional workforce certificated by the civil aviation authority in every country. These kinds of certificated workforce include pilots for civil air transport and general aviation, air traffic controllers, dispatchers, maintenance engineers, and ground machinists in Taiwan. In particular, the manpower requirement for pilots represents the scale of the air transport market in one country and concerns the certification affairs operated by the civil aviation authority. An appropriate workforce demand forecast model can assist the authority in realizing the future development of the whole aviation industry. This study aims to analyze the demand trends of airmen in Taiwan and proposes forecast models to predict the future manpower requirement of pilots in civil air transport. Based on limited samples published in the official reports, this study applied the grey theory to construct GM(1,1) models to predict all pilots, pilots in international airlines, and pilots in regional airlines. These models were evaluated as of good forecasting abilities. The major contribution of this work is to propose a pilot prediction model without a large number of samples. We also found that GM (1, N) model would be more suitable for pilot manpower demand forecast in fleets expansion than other GM models.

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## INTRODUCTION

Under the trend of globalization, transportation of passenger and cargo bloom rapidly. Aircraft are the main means of transport, and are especially important for passenger transport. Take Taiwan for example, due to the world trade and cross trait transportation rise, Taiwan Taoyuan International Airport passenger (enplaned+depland), aircraft movements (arriving+departing), the number of freight (tonnes) experienced a steady growth every year (Fig. 1 - Fig. 3). In 2014, the number of passengers has reached 36 million, representing a growth rate of 66.84% from 2004 to 2014.

With the growing demand for air transportation, the airline industry has to expand its fleets. As the scale of fleets grows, need for air men also rises. Since the training and development of air men take lot of time and are very costly. Human resources planning for air men is essential. On the demand basis, aviation authority and airline companies can also have a more precise picture about the manpower gap and take necessary actions in advance.

The purpose of this study is to build a model for pilot forecasting. Using the yearly data of certified airline pilots and aircrafts of Taiwan, this work tries to build a pilot forecasting model based on airline crafts. We built both GM (1,1) and GM(1,N) model. The GM (1,1) model is for time series forecast of pilot manpower. The GM (1,N) considers different types and

amounts of aircrafts for pilot manpower forecasting.

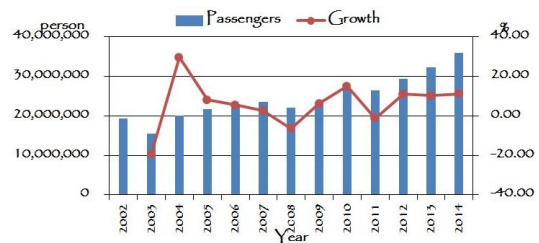


Fig. 1 . Growth in Taiwan Taoyuan interational airport passengers  
Source: Airport-information

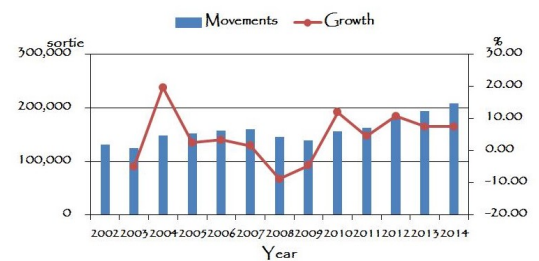


Fig. 2 . Growth in Taiwan Taoyuan interational airport aircrafts movements  
Source: Airport-information

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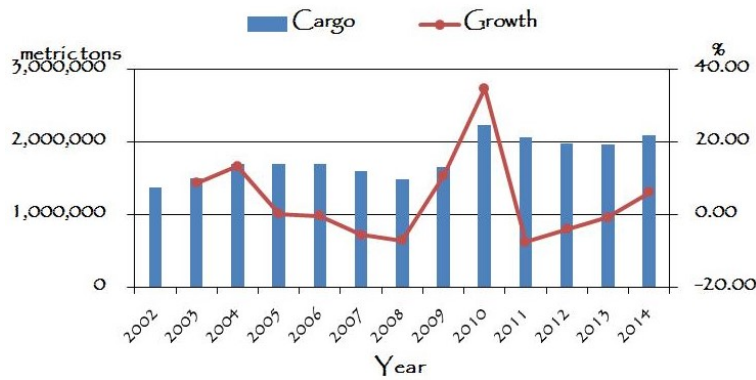


Fig. 3 . Growth in Taiwan Taoyuan interational airport cargo  
Source: Airport-information

**LITERATURE REVIEW**

Grey system theory was proposed by Deng [1]. Akay [2] argued that GPRM approach is an efficient model because of the high accuracy and applicability in the case of limited data. Besides, GPRM requires little computational effort [2]. Chang [3] also stated that grey model is an effective approach for data analysis with small samples. Because of its ease of use, grey system theory has been successfully applied in various domains [4, 5, 11]. The major function of manpower forecasting is to understand the future supply and demand of the labor market, and estimate the quantity of human resources required in a time period [6, 12]. Many approaches are applied to manpower forecasting. Time series is one of the important methods of human resources demand forecasting. Zhou explained that time series refers to the string of numbers in chronological order, and time series forecasting is time series data, using statistical models to deduce the future. Babbie [7] pointed that time series analysis can be used to show long-term trends, and provide an explanation of the trend. Therefore, this study will be used as a time series analysis of grey forecast to predict pilot manpower demand assessment.

**METHODOLOGY**

A Grey system is one with either incomplete or undetermined information. Just as in the real world, black can represent a lack of information, whereas white represents a plethora of information. Thus, the region between black and white is called grey. The formulation of prediction models is one of the major applications of Grey theory. The idea of Grey prediction stems from discrete differential equations. It is more suitable for forecasting when data are limited. Grey prediction models are normally classified according to differential order and involve sequences. They are represented as, GM (h,N), where h represents the order and N is the number of sequences

involved. For example, a GM (1,1) model denotes a first order discrete differential equation, for a single sequence, while a GM (1,N) model denotes a first order discrete differential equation, for N sequences.

This study tries to build a prediction model for airline pilots. The concepts for the Grey processes used in this study are described as below.

**The Grey Prediction Model GM (1,1)**

Grey prediction models are based on the concept of discrete differential equations. Normally differential equations are used to describe the essence of development, using continuous functions, not discrete sequence data, because of its non-differentiability. Grey differential equations construct a series of theorems, using discrete differential equations. Suppose that  $X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n))$  is a non-negative sequence of raw data. Its first-order accumulative generation operator (1-AGO) sequence is defined as  $X^{(1)} = (x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(n))$ , where

$$x^{(1)}(k) = \sum_{i=1}^k x^{(0)}(i), k = 1, 2, \dots, n \quad (1)$$

$X^{(1)}$  is not a Grey differential sequence, because it cannot satisfy the arithmetic horizontal mapping relationships [10]. Equation (2) provides a generated mean value sequence,  $Z^{(1)} = (z^{(1)}(2), z^{(1)}(3), \dots, z^{(1)}(n))$ , for consecutive neighbors of the sequence  $X^{(1)}$ . The sequence,  $Z^{(1)}$ , can be treated as a Grey differential sequence, because its Grey derivative, replaced by  $x^{(0)}(k)$ , satisfies the aforementioned mapping relationships. As with the continuous type, in Equation (3), Equation (4) is a Grey differential equation: the so-called GM(1,1) model.

$$z^{(1)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k - 1), k = 2, 3, \dots, n \quad (2)$$

$$\frac{dx^{(1)}t}{dt} + ax^{(1)}t = b \quad (3)$$



$$x^{(0)}(k) + az^{(1)}(k) = b, k = 2, 3, \dots, n \quad (4)$$

The parameters of Model GM(1,1), i.e. a and b, can be estimated by the least square method, as follows:

$$[\hat{a}, \hat{b}]^T = (B^T B)^{-1} B^T Y_N,$$

where

$$Y_N = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}, B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix} = \begin{bmatrix} -0.5(x^{(1)}(2) + x^{(1)}(1)) & 1 \\ -0.5(x^{(1)}(3) + x^{(1)}(2)) & 1 \\ \vdots & \vdots \\ -0.5(x^{(1)}(n) + x^{(1)}(n-1)) & 1 \end{bmatrix}$$

Using this estimation for a and b, Equation (5) is the solution (or the so-called time response sequence) of the GM (1,1) model, where  $x^{(1)}(0)$  is replaced by  $x^{(0)}(1)$ . The restored values of  $X^{(0)}$  can be obtained, through the inverse accumulated generating operation (IAGO), described in Equation (6). Equations (5) and (6) are used to forecast the development of sequence,  $X^{(0)}$ .

$$\hat{x}^{(1)}k + 1 = (x^{(0)}(1) - \frac{\hat{b}}{\hat{a}})e^{\hat{a}k} + \frac{\hat{b}}{\hat{a}}, k = 1, 2, \dots, n \quad (5)$$

$$\hat{x}^{(0)}k + 1 = (\hat{x}^{(1)}(k + 1) - \hat{x}^{(1)}(k)), k = 1, 2, \dots, n \quad (6)$$

**The Grey Prediction Model GM (1,N)**

Following the derivation of the GM(1,1) model, a GM(1,N) model, is a multivariable Grey model. Suppose that there is a sequence system,  $X^{(0)}$ , with N sequences of raw data,  $X_j^{(0)}, j = 1, 2, \dots, N, X_j^{(0)} = (x_j^{(0)}(1), x_j^{(0)}(2), \dots, x_j^{(0)}(n))$ .  $X_1^{(0)}$ , therefore, represents the sequence of system characteristics, while  $X_j^{(0)}, j = 2, \dots, N$  are sequences of relevant factors. The prediction model must be constructed for the sequence of system characteristics, in terms of sequences of relevant factors.

The 1-AGO sequences and the generated mean value sequences are defined as  $X_j^{(1)}$  and  $Z_j^{(1)}, j = 1, 2, \dots, N$ , respectively. Their operations are the same as those of the GM(1,1) model, in Equations (7) and (8). Equation (9) is called a GM(1,N) Grey differential equation, where -a is called the development coefficient of the system and bi the driving coefficient.

$$x_j^{(1)}(k) = \sum_{i=1}^k x_j^{(0)}(i), k = 1, 2, \dots, n, j = 1, 2, \dots, N \quad (7)$$

$$Z_j^{(1)}(k) = 0.5x_j^{(1)}(k) + 0.5x_j^{(1)}(k - 1), k = 2, 3, \dots, n : \quad (8)$$

$$j = 1, 2, \dots, N$$

$$x_1^{(0)}(k) = -az_1^{(1)}(k) \sum_{i=2}^N b_i x_i^{(1)}(k), k = 2, 3, \dots, n, \quad (9)$$

The coefficients in Equation (9) can be estimated by the least square method, as follows:

$$[\hat{a}, \hat{b}_2, \hat{b}_3, \dots, \hat{b}_N]^T = (B^T B)^{-1} B^T Y_N$$

where

$$Y_N = \begin{bmatrix} x_1^{(0)}(2) \\ x_1^{(0)}(3) \\ \vdots \\ x_1^{(0)}(n) \end{bmatrix}, \text{ and } B = \begin{bmatrix} -z_1^{(1)}(2) & x_2^{(1)}(2) & \dots & x_N^{(1)}(2) \\ -z_1^{(1)}(3) & x_2^{(1)}(3) & \dots & x_N^{(1)}(3) \\ \vdots & \vdots & \dots & \vdots \\ -z_1^{(1)}(N) & x_2^{(1)}(N) & \dots & x_N^{(1)}(N) \end{bmatrix}$$

Using this estimation for the coefficients, Equation (10) is the time response sequence of the GM (1,N) model, where  $x_1^{(0)}(1)$  replaces  $x_1^{(1)}(0)$ . The restored values of  $X_1^{(0)}$  can be obtained via the IAGO and are expressed as Equation (11). However, most studies employed a different GM (1,N) simulation expression, as given by Equation (12), to estimate  $X_1^{(0)}$  [17]. This study follows this general method, also.

$$\hat{x}_1^{(1)}(k + 1) = [x_1^{(0)}(1) - \frac{1}{\hat{a}} \sum_{i=2}^N \hat{b}_i x_i^{(1)}(k + 1)]e^{-\hat{a}k} + \frac{1}{\hat{a}} \sum_{i=2}^N \hat{b}_i x_i^{(1)}(k + 1) \quad (10)$$

$$\hat{x}^{(0)}(k + 1) = \hat{x}_1^{(1)}(k + 1) - \hat{x}_1^{(1)}(k) \quad (11)$$

$$\hat{x}_1^{(0)}(k) = -\hat{a}z_1^{(1)}(k) + \sum_{i=2}^N \hat{b}_i x_i^{(1)}(k), k = 2, 3, \dots, n \quad (12)$$

**Data**

Data are available from Taiwan Civil Aeronautics Administration (CAA) statistical database ([http://www.wto.org/english/res\\_e/statis\\_e/statis\\_e.htm](http://www.wto.org/english/res_e/statis_e/statis_e.htm)). The database contains 28 airline companies data of aircrafts (with different type) and air men. This study retrieved data from year 2001 to 2013 for model building.

**RESULTS AND DISCUSSION**

This research tries to build pilot forecasting model based on time serial (GM (1,1)) and on number of aircrafts (GM (1,N)). Two international airlines of Taiwan (China airlines (CI) and EVA Airways (BR)) were chosen as sample for analysis. Data of China airlines and EVA Airways are summarized in Table 1. The results are as follows:



TABLE 1  
DATA OF CHINA AIRLINES AND EVA AIRWAYS

Year	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014
CI Pilot	830	824	889	922	1022	1064	1064	994	944	927	957	985	995	1014
CI Aircraft														
A300-600R	8	12	12	12	6	1	0	0	0	0	0	0	0	0
A330-300	0	0	0	3	8	12	16	17	15	18	19	22	23	24
A340-300	5	5	7	7	7	7	6	6	6	6	6	6	6	6
B737-800	9	11	11	11	11	12	11	11	10	10	10	10	14	16
B747-200	5	1	0	0	0	0	0	0	0	0	0	0	0	0
B747-400	16	19	22	14	15	14	14	12	12	12	12	12	12	12
B747-400F	0	0	0	15	18	20	20	20	20	19	19	21	21	21
BR Pilot	586	617	655	683	679	700	634	637	634	637	712	787	821	933
BR Aircraft														
A318-112	0	0	0	0	0	0	0	0	0	0	0	0	1	2
A321-211	0	0	0	0	0	0	0	0	0	0	0	3	6	12
A330-200	0	0	2	6	10	11	11	11	11	11	11	11	11	11
A330-300	0	0	0	0	0	0	0	0	0	0	0	3	3	3
B747-400	7	8	8	3	6	6	6	6	6	6	6	6	5	5
B747-400COMBI	6	6	6	3	2	2	2	0	0	0	0	0	0	0
B747-400EBC	0	0	0	2	2	2	2	2	2	2	2	0	0	0
B747-400F	0	0	0	3	3	3	3	3	3	3	3	3	3	3
B747-400SF	0	0	0	0	0	0	0	0	0	0	0	6	6	5
B747-45E	4	4	4	7	5	5	5	5	5	5	5	1	0	0
B767-200	4	4	4	4	0	0	0	0	0	0	0	0	0	0
B767-300ER	2	2	2	2	2	0	0	0	0	0	0	0	0	0
B777-300ER	0	0	0	0	2	4	8	11	14	15	15	15	15	18
MD-11F	11	11	11	11	10	10	9	8	8	8	8	6	6	6
MD-90	0	0	0	0	0	0	0	0	0	0	0	2	7	4

**GM(1,1) Model**

Model GM (1,1) used China airlines(CI) and EVA Airways(BR) pilots data to generate a model for this series, with exponential development. The first step was the creation of the first-order accumulative generation operator sequence, using Equation (1), and its generated mean value sequence, in terms of Equation (2).

CI:

$$X^{(0)} = (830, 824, 889, 922, 1022, 1064, 1064, 994, 944, 927, 957, 985, 995, 1014)$$

$$X^{(1)} = (830, 1654, 2543, 3465, 4489, 5551, 6615, 7609, 8553, 9480, 10437, 11422, 12417, 13431)$$

$$Z^{(1)} = (1242, 2098.5, 3004, 3976, 5019, 6083, 7112, 8081, 9016.5, 9958.5, 10929.5, 12924)$$

BR:

$$X^{(0)} = (586, 617, 655, 683, 679, 700, 634, 637, 634, 637, 712, 787, 821, 933)$$

$$X^{(1)} = (586, 1203, 1858, 2541, 3220, 3920, 4554, 5191, 5825, 6462, 7174, 7961, 8782, 9715)$$

$$Z^{(1)} = (894.5, 1530.5, 2199.5, 2880.5, 3570, 4237, 4872.5, 5508, 6143.5, 6818, 7567.5, 8371.5, 9248.5)$$

The estimated parameters were calculated, by the least square method, as follows:

CI:

$$\begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = \begin{bmatrix} -0.00735 \\ 917.649 \end{bmatrix}$$

BR:

$$\begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = \begin{bmatrix} -0.02584 \\ 575.3241 \end{bmatrix}$$

The time response sequence of the GM (1,1) model, for prediction of CI and BR pilots, is represented in Equation (13). The series for the estimated 1-AGO can then be calculated, as follows:

CI:

$$\hat{x}^{(1)}(k+1) = \left(x^{(0)}(1) + \frac{917.649}{0.00735}\right) e^{0.00735k} - \frac{0.00735}{917.649} \quad (13)$$

BR:

$$\hat{x}^{(1)}(k+1) \left(x^{(0)}(1) + \frac{575.3241}{0.02584}\right) e^{0.02584k} - \frac{0.02584}{575.3241}$$

CI:

$$\hat{X}^{(1)} = (830, 1757.153, 2691.146, 3632.03, 4579.856, 5534.673, 6496.536, 7465.494, 8441.601, 9424.909, 10415.47, 11413.34, 12418.57)$$

BR:  
 $\hat{X}^{(1)} = (586, 1184.163, 1797.985, 2427.876, 3074.257, 3737.56, 4418.227, 5116.712, 5833.484, 6569.019, 7323.81, 8098.36, 8893.187)$

Using the IAGO represented in Equation (6), it was possible to obtain the predicted value for each year. Table 2

shows the residual results for the CI pilots and BR pilots. The largest absolute value of CI residuals, 12.52%, occurs in 2002, while the absolute value of residuals in 2014 was only 0.13%. The largest absolute value of BR residuals, 15.47%, occurs in 2010, while the absolute value of residuals in 2012 was only 1.58%.

TABLE 2  
 RESIDUAL RESULTS OF THE CI AND BR GM (1, 1) MODEL

Year	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014
CI	12.52%	5.06%	2.05%	7.26%	10.26%	9.60%	2.52%	3.40%	6.07%	3.51%	1.31%	1.03%	0.13%
BR	3.05%	6.29%	7.78%	4.80%	5.24%	7.36%	9.65%	13.06%	15.47%	6.01%	1.58%	3.19%	12.58%

Source: Compiled by this study

**GM (1, N) Model**

This section will display the GM (1, N) model results.

three data of aircraft, because the data are less. Table 3 shows the aircraft types are adoption about GM(1,5), GM(1,8). Table 3 shows CI airline has seven aircraft types. The first step in the construction of the GM (1,8) model is retrieval of the data for CI pilots. The first-order accumulative generation operator sequences and the generated mean value sequences were calculated using Equations (7) and (8).

**GM (1, N) of CI Pilots**

This section will construct GM (1,N) model of CI airline pilots prediction. In this study, N representative aircraft type, GM (1,8) has eight aircraft types and GM (1,5) is reduced to

TABLE 3  
 GM(1,5) AND GM(1,8) ADOPTION THE DATA OF AIRCRAFT TYPES

Model	GM(1,5)	GM(1,8)
Including Aircraft Types	A340-300, B737-800, B747-m400, B747-400F	A300-600R, A330-300, A340-300, B737-800, B747-200, B747-400, B747-400F

Source: Compiled by this study

CI GM(1,8) Model:

$$X^{(0)} = \begin{bmatrix} x_{pilot}^{(0)} \\ x_{A306}^{(0)} \\ x_{A333}^{(0)} \\ x_{A343}^{(0)} \\ x_{B738}^{(0)} \\ x_{B742}^{(0)} \\ x_{B744}^{(0)} \\ x_{B744F}^{(0)} \end{bmatrix} = \begin{bmatrix} 830 & 824 & 889 & 922 & 1022 & 1064 & 1064 & 994 & 944 & 927 & 957 & 985 & 995 & 1014 \\ 8 & 12 & 12 & 12 & 6 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 8 & 12 & 16 & 17 & 15 & 18 & 19 & 22 & 23 & 24 \\ 5 & 5 & 7 & 7 & 7 & 7 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 \\ 9 & 11 & 11 & 11 & 11 & 12 & 11 & 11 & 10 & 10 & 10 & 10 & 14 & 16 \\ 5 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 16 & 19 & 22 & 14 & 15 & 14 & 14 & 12 & 12 & 12 & 12 & 12 & 12 & 12 \\ 0 & 0 & 0 & 15 & 18 & 20 & 20 & 20 & 20 & 19 & 19 & 21 & 21 & 21 \end{bmatrix}$$

$$X^{(1)} = \begin{bmatrix} x_{pilot}^{(1)} \\ x_{A306}^{(1)} \\ x_{A333}^{(1)} \\ x_{A343}^{(1)} \\ x_{B738}^{(1)} \\ x_{B742}^{(1)} \\ x_{B744}^{(1)} \\ x_{B744F}^{(1)} \end{bmatrix} = \begin{bmatrix} 830 & 1654 & 2534 & 3465 & 4487 & 5551 & 6615 & 7609 & 8553 & 9480 & 10437 & 11422 & 12417 & 13431 \\ 8 & 20 & 32 & 44 & 50 & 51 & 51 & 51 & 51 & 51 & 51 & 51 & 51 & 51 \\ 0 & 0 & 0 & 3 & 11 & 23 & 39 & 56 & 71 & 89 & 108 & 130 & 153 & 177 \\ 5 & 10 & 17 & 24 & 31 & 38 & 44 & 50 & 56 & 62 & 68 & 74 & 80 & 86 \\ 9 & 20 & 31 & 42 & 53 & 65 & 76 & 87 & 97 & 107 & 117 & 127 & 141 & 157 \\ 5 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 \\ 16 & 35 & 57 & 71 & 86 & 100 & 114 & 126 & 138 & 150 & 162 & 174 & 186 & 198 \\ 0 & 0 & 0 & 15 & 33 & 53 & 73 & 93 & 113 & 132 & 151 & 172 & 193 & 214 \end{bmatrix}$$



$$z_{pilot}^{(1)} = (1242, 2098.5, 3004, 3976, 5019, 6083, 7112, 8081, 9016.5, 9958.5, 10929.5, 11919.5, 12924)$$

From the definition of Equation (9), the relative parameters can be estimated by the least square method, as follows:

$$\begin{bmatrix} \hat{a} \\ \hat{b}_2 \\ \hat{b}_3 \\ \hat{b}_4 \\ \hat{b}_5 \\ \hat{b}_6 \\ \hat{b}_7 \\ \hat{b}_8 \end{bmatrix} = \begin{bmatrix} 1.65774 \\ -1.38915 \\ -2.78227 \\ -36.2933 \\ 9.97387 \\ 74.06231 \\ 75.06872 \\ 43.22582 \end{bmatrix}$$

The GM (1,8) model, for the prediction of CI pilots, is shown as Equation (14). The predicted value for each year can be obtained. Table 4 shows the residual results of GM (1,5)

and GM (1,8) model. The largest absolute value of GM (1,8) residuals, 3.78%, occurred in 2010, while the absolute value of residuals in 2014 was only 0.01%. The residual results of GM (1,5), the largest absolute value of residuals, 8.32%, occurred in 2002, while the absolute value of residuals in 2009 was only 0.07%.

$$\begin{aligned} \text{GM(1,8)} \\ \hat{x}_{pilot}^{(0)}(k) = & -1.65774z_{pilot}^{(1)}(k) - 1.38915x_{A300-600R}^{(1)}(k) \\ & - 2.78227x_{A330-300}^{(1)}(k) - 36.2933x_{A340-300}^{(1)} \\ & + 9.97387(k)x_{B737-800}^{(1)}(k) + 74.06231x_{B747-200}^{(1)}(k) \\ & + 75.06872x_{B747-400}^{(1)}(k) + 43.22582x_{B474-400F}^{(1)}(k) \end{aligned} \quad (14)$$

$$\begin{aligned} \text{GM(1,5)} \\ \hat{x}_{pilot}(k) = & -2.3739634z_{pilot}^{(1)}(k) + 4.81948408x_{A340-300}^{(1)}(k) \\ & + 140.402233x_{B737-800}^{(1)}(k) - 130.65382x_{B747-400}^{(1)} \\ & + 67.2063867x_{B474-400F}^{(1)}(k) \end{aligned}$$

TABLE 4  
RESIDUAL RESULTS OF GM (1, 5) AND GM (1,8) MODEL

Year	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014
GM(1,5)	8.32%	8.32%	8.32%	8.32%	8.32%	8.32%	8.32%	8.32%	8.32%	8.32%	8.32%	8.32%	8.32%
GM(1,8)	0.29%	0.29%	0.29%	0.29%	0.29%	0.29%	0.29%	0.29%	0.29%	0.29%	0.29%	0.29%	0.29%

Source: Compiled by this study

**GM (1, N) of BR Pilots**

This section will construct GM (1,N) model of BR airline pilots prediction. BR airline has fifteen aircraft types (Table 1). The first step in the construction of the GM (1,16) model is retrieval of the data for BR pilots. And construction of GM (1,8) and GM (1,11) respectively. Table 5 shows the aircraft types are adoption about GM (1,8), GM (1,11) and GM (1,16).

Table 6 shows the residual results of BR prediction. Table 6 shows the prediction of GM (1,16) that is very inaccurate.

Therefore, we have not used less data of aircraft to construct GM (1,N) model, respectively, GM(1,8) and GM(1,11). Table 5 shows the aircraft types are adoption about GM (1,8), GM (1,11) and GM (1,16). GM (1,8) largest absolute value of residuals, 7.27%, occurred in 2005, while the absolute value of residuals in 2008 was only 0.56%. GM (1,11) the largest absolute value of residuals, 1.29%, occurred in 2010, while the absolute value of residuals in 2002, 2003, 2004, 2005 and 2006 was 0.00%.

TABLE 5  
GM (1,8), GM (1,11) AND GM (1,16) ADOPTION THE DATA OF AIRCRAFT TYPES

Model	GM(1,8)	GM(1,11)	GM(1,16)
Including Aircraft Types	A330-200, B747-400, B747-400COMBI, B747-400EBC, B747-400F, B747-45E, MD-11F	A330-200, B747-400, B747-400COMBI, B747-400EBC, B747-400F, B747-45E, B767-200, B767-300ER, B777-300ER, MD-11F	A318-112, A321-211 A330-200, A330-300 B747-400, B747-400COMBI, B747-400EBC, B747-400F, B747-400SF, B747-45E, B767-200, B767-300ER, B777-300ER, MD-11F, MD-90

Source: Compiled by this study

TABLE 6  
RESIDUAL RESULTS OF GM (1,8), GM (1,11) AND GM (1,16) MODEL

Year	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014
GM(1,8)	4.06%	2.36%	3.98%	7.27%	2.30%	2.87%	0.56%	2.68%	4.96%	4.81%	1.60%	3.90%	1.84%
GM(1,11)	0.00%	0.00%	0.00%	0.00%	0.00%	0.27%	0.56%	0.74%	1.29%	1.07%	0.52%	0.99%	0.44%
GM(1,16)	229.95%	337.99%	540.61%	712.57%	849.56%	1109.72%	1270.22%	1443.54%	1602.28%	1587.02%	1460.93%	1424.74%	965.96%

Source: Compiled by this study

**Validity of Model and Result Comparisons**

Lewis [8] proposed the two most used statistics, the mean squared error (MSE) and the mean absolute percentage error (MAPE), to evaluate the precision and accuracy of forecasting models. Leitch and Tanner [9] proposed a commonly used statistic, the Theil’s U. If  $x(i)$  is the original data and  $\hat{x}(i)$  is the prediction value, then these three indices are defined as Equations (19) to (21), for n observations.

$$MSE = \frac{1}{n} \sum_{i=1}^n (x(i) - \hat{x}(i))^2 \tag{15}$$

$$MAPE = \frac{100}{n} \sum_{i=1}^n \frac{|x(i) - \hat{x}(i)|}{x(i)} \tag{16}$$

$$U = \sqrt{\frac{\sum_{i=1}^n (x(i) - \hat{x}(i))^2}{\sum_{i=1}^n (x(i))^2}} \tag{17}$$

The MSE is the average of the squared forecasting errors. A good prediction model is supposed to have small MSE, but will be different, when using various units of the same data. The MAPE method uses the average value of the forecasting error, expressed as a percentage of the relevant observed value, regardless of whether the error is positive or negative. The Theil’s U method can be viewed as the root-mean-squared error of a forecast, divided by a naive forecast of no change. Table 7 shows the evaluation criteria for MAPE and Theil’s U statistics. The closer to zero the index, regardless of the index, the higher is the predictive ability of a model.

TABLE 7  
RESIDUAL RESULTS OF GM (1,8), GM (1,11) AND GM (1,16) MODEL

MAPE	Forecast Potential	Theil’s U	Comparison with no-change forecast
<10%	Very good	U = 0	Perfect
10% ~ 20%	Good	U < 1	Lower Errors
20% ~ 30%	Reasonable	U = 1	Equal
>30%	Inaccurate	U > 1	Higher Errors

Source: [9, 10]

Table 8 shows a comparison of the previously mentioned evaluation indices, for the CI and BR GM (1, N) models. In CI’s case, the GM (1,8) model demonstrates a lower minimum absolute residual than the GM (1,1) and GM (1,5). The GM (1, 1) model demonstrates a higher maximum absolute residual than the GM (1,5) and GM (1,8) model. The MSEs of the CI’s three models are 3623.346, 1.17E+03 and 2.53E+02. The forecast potential of the CI’s models is regarded as very good, at 4.98% 2.92% and 1.15% of MAPE. CI’s three models demonstrate lower errors than the naive forecast, since their Theil’s U statistics are all less than 1.0. The GM (1,8) model also has a

higher predictive ability than the GM (1,1) and GM (1,5). In BRs case, the GM (1,11) model has best prediction result. The MSEs of the BR’s models are 3557.387, 6.60E+02, 2.02E+01 and 6.62E+07. The forecast potential of the BRs models in addition to the GM (1, 16), is regarded as very good, at 7.39% 3.32% and 0.45% of MAPE. BRs models demonstrate lower errors than the naive forecast, since their Theil’s U statistics are less than 1.0 except the GM (1, 16) that is greater than 1. The GM (1,11) model has a higher predictive ability than the GM (1, 1), GM (1, 8) and GM (1, 16).



TABLE 8  
PREDICTION EVALUATION FOR CI AND BR GM MODELS

Model	Minimum Absolute Residual		Maximum Absolute Residual		MSE	MAPE		Theil's U	
	Year	Residual	Year	Residual	Value	Forecast Potential	Value	Comparison with no-change forecast	
CI GM(1,1)	2014	0.13%	2002	12.52%	3623.346	4.98%	Very good	0.061957	Lower Errors
CI GM(1,5)	2009	0.07%	2002	8.32%	1.17E+03	2.92%	Very good	0.000617	Lower Errors
CI GM(1,8)	2014	0.01%	2010	3.87%	2.53E+02	1.15%	Very good	0.000134	Lower Errors
BR GM(1,1)	2012	1.58%	2010	15.47%	3557.387	7.39%	Very good	0.084261	Lower Errors
RR GM(1,8)	2008	0.56%	2005	7.27%	6.60E+02	3.32%	Very good	0.000659	Lower Errors
BR GM(1,11)	2002-2006	0%	2010	1.29%	2.02E+01	0.45%	Very good	0.000020	Lower Errors
RR GM(1,16)	2011	1587.02%	2002	229.95%	6.62E+07	1041.16%	Inaccurate	66.0274	Higher Errors

Figure 4 illustrates the trends in the CI pilots, for three prediction models. The series for model GM (1,N) is indeed closer to the raw data than that for model GM (1,1). During the years 2005 to 2007, GM (1,1) maintained its trend of exponential growth, revealing distinguishable differences with the forecast of model GM (1,N).

The GM (1,16) model prediction is very inaccurate and cannot show complete chart.

Figure 5 illustrates the trends in the BR pilots, for three

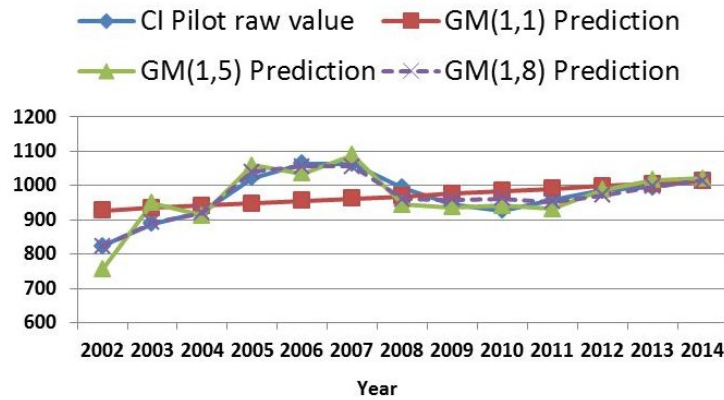


Fig. 4 . CI Pilots in real values and the predicted values of the three models

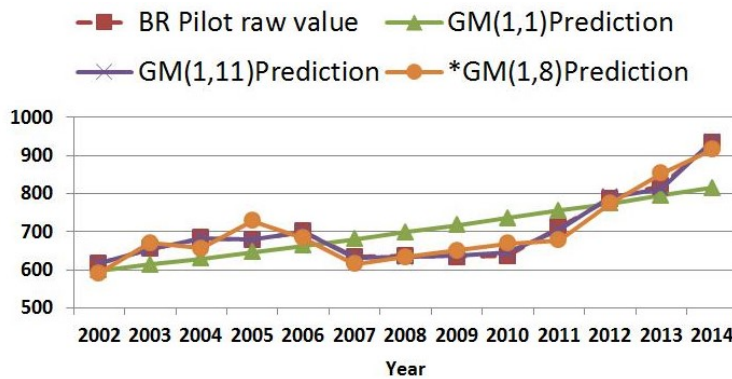


Fig. 5 . BR Pilots in real values and the predicted values of the three models

## CONCLUSION AND FUTURE RESEARCH

In this study, we used the grey forecast to predict manpower demand of pilots. In general, researchers use the grey relational analysis (GRA) to filter factors before building the GM (1,N) model. But in this study, the type of data is not allowed to use GRA. It has to determine the best prediction of any combined data. We constructed GM models, CI and BR's GM (1,N) models all provided with much higher predictive ability, and the performance of GM (1,N) was better than GM (1,1) model. The major contribution of this work is to propose pilot prediction model without large number of samples. We also found that GM (1,N) model would be more suitable for pilot manpower demand forecast in the case of fleets expansion than other GM models. Besides, forecasting model varied when considering the difference of pilot disposing policy. From CI and BR GM (1,N) model, we found that CI with all aircraft

types in GM (1,N) model has very good MAPE forecast potential. While the same approach applied to BR pilot prediction did not fit well. The result of MAPE forecast potential is relatively low when GM (1,N) model with all aircraft types of BR is put into the model. We can infer that CI airline and BR airline dispose their pilots differently, and each airline company has its unique pilot staffing and development policy which leads to different forecasting model. From the research data, we can't know how many pilots in each aircraft. When we use Grey Forecast to predict the number of pilots, we must select the aircraft models' combination first. Although the relationships between airline's aircraft and their pilots may change slowly over time, the Grey theory procedure, used in this study, should provide a reliable prediction at any time. It is expected that data from this industry might also be used for the construction of useful prediction models.

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— This article does not have any appendix. —