International Journal of Technology and Engineering Studies
volume 4 issue 4 pp. 132-142 doi: https://dx.doi.org/10.20469/ijtes.4.10002-4

An Identification Method of Squareness Errors Based on Volumetric Error Model in Machine Tools

Xuemin Zhong*
School of Mechanical Science and Engineering, Huazhong University of Science and Technology, Wuhan, China

Hongqi Liu
School of Mechanical Science and Engineering, Huazhong University of Science and Technology, Wuhan, China

Hao Chang
School of Mechanical Science and Engineering, Huazhong University of Science and Technology, Wuhan, China

Bin Li
School of Mechanical Science and Engineering, Huazhong University of Science and Technology, Wuhan, China

Abstract: This paper proposes a new method to measure and identify the squareness errors of machine tools considering the volumetric accuracy based on the volumetric error model. The volumetric error model based on screw theory is proposed, and then the identification method for the squareness errors is deduced in theory considering a 3-axis horizontal machine tool. Experiments with the measuring method and other traditional measuring methods have verified that the three identified squareness errors follow the measured squareness within the accuracy range. Moreover, the six face diagonal errors are calculated with the identified squareness errors and other squareness errors in different methods compared with the measured face diagonal errors. It shows that compared with the traditional measurement of squareness errors, the proposed method shows more effectiveness in the volumetric error evaluation process.

Keywords: *Volumetric error model, squareness error, error identification, laser interferometer*

Received: 04 April 2018; ***Accepted:*** 18 July 2018; ***Published:*** 22 August 2018

I. INTRODUCTION

The accuracy performance is significant for the precision machine tool nowadays and to measure, model and predict the total errors of the machine tool is important so that the total error can be reduced or compensated in the machine tool. The paper researches on the identification method of squareness errors in volumetric error modeling and evaluation process. The volumetric error represents the overall error of the machine tool which is mainly caused by geometric errors [1]. The volumetric error model needs to be established so that the geometric errors are all mapped to the volumetric error. Then the geometric errors can be analyzed based on the volumetric error model. On the other hand, the volumetric error model is used to calculate the volumetric error and then volumetric error compensation is implemented in the machine tool so that overall error of the machine tool is decreased [2]. Volumetric error models for the machine tool can be proposed by many methods, such as the Denavit-Hartenberg (D-H) transformation matrix method [3], differential transformation matrix method [4], exponential formulas method [5] and screw theory method [2, 6]. The forward kinematics in the screw theory is used to represent the error motions and establish the error models in the paper for its direct physical meaning and no many local coordinate systems needed. From the volumetric error model, the relationship of all the geometric errors and the effects of each parameter on the volumetric error can be analyzed by error sensitivity analysis [7]. The analysis results show the squareness errors have relatively

*Correspondence concerning this article should be addressed to Xuemin Zhong, School of Mechanical Science and Engineering, Huazhong University of Science and Technology, Wuhan, China. E-mail: zhongxuemin@hust.edu.cn

large effects than positioning and straightness errors and cannot be ignored [8]. To acquire the precise geometric errors especially the squareness errors is significant to improve the machine tool accuracy.

The methods of the measurement for the geometric errors are classified as the direct methods and indirect methods [9]. The direct methods take the measurement of the geometric errors directly while the indirect methods take the measurement of the superposed errors. In the indirect methods, the geometric errors are usually identified with certain identification models. Because some geometric errors cannot be measured directly and it is usually with low efficiency using the direct methods, the indirect methods are more popularly researched and presented. In most cases, the geometric errors are coupled together and the Abbe errors exist, the measured end errors are in fact the overall errors, the indirect methods should be presented correspondingly [10, 11].

Different indirect methods based on various measuring instruments are presented in previous literature. [12] designed the measurement patterns of the 11 location errors on machining the cubic workpiece. The laser displacement sensor is used and identification model is proposed for each location error. The double ball bar is used to design certain measuring types and geometric errors concerned are measured out by proposed identification models [13]. The measuring methods are proposed by using the laser interferometer and all geometric errors of the 3-axis machine are identified in an effective way [14]. The R-test for the rotary axis is adopted to acquire position dependent errors and position independent errors associated with each rotary axis [15, 16, 17]. A probe ball bar is also applied to present methods to acquire the errors of a rotary axis [18, 19]. The geometric errors are also measured and identified by methods using a laser tracker [20, 21, 22]. The measuring method of squareness errors are also adopted using the ball bar [23], laser interferometer [24] and a mechanical square with an indicator [11]. The face diagonal length measurement method is also presented in ISO 230-6 [25]. The new 4 body diagonal length measurement method with a single properly sized artefact is also proposed to measure the 3 squareness errors effectively [26]. However, the ball bar is applied to take measurement of squareness errors conveniently, yet it usually measures the local squareness errors and not all machine tools are suitable to carry out three circle trajectories to measure all the squareness errors using the ball bar. The laser interferometer with the mirror set costs a very long time for aligning laser with high skill and is rarely used in most cases. The face or body diagonal length method measures the length that is redundant for environment and other geometric errors coupled are ignored. Other traditional methods are usually not with high accuracy and would not be proper for volumetric error evaluation of machine tools. The squareness errors are usually seen as the angle errors or the displacement errors modeled in the overall errors [27, 28, 29]. However, the other angle errors of the translational axis would usually have effects on the measurement trajectory of the squareness measurement, which is caused by Abbe offset [24]. In most cases, this factor is ignored in the measurement of squareness errors and in the volumetric error compensation process [30, 31]. The new squareness error measuring and identification method considering other coupled geometric errors and volumetric error is researched in the paper. The study would give new ideas and methods in the enhancement of machine tool accuracy.

The other sections in the paper is shown in the followings: In section 2, a horizontal machine tool is introduced to give clarifications, kinematic chains and geometric errors are introduced and the volumetric error model using the screw theory is presented. In section 3, the measurement method of the squareness error is presented as well as the identification method, followed by implementing the measuring experiment to verify the method in Section 4. At last in Section 5, conclusions are therefore presented.

II. VOLUMETRIC ERROR MODELING

A. *Geometric Errors Descriptions*

As is drawn in Fig 1, the investigated 3-axis horizontal machine tool is composed of a bed, z-axis guideway, y-axis guideway, x-axis guideway, worktable, vertical column, spindle system and cutting tool. The global base coordinate frame is set fixedly to the bed with each axis direction matches the machine axis direction correspondingly. Three translational axes are concerned and there are 21 geometric error parameters including the six PDGEs of each axis and three PIGEs between every two axes. The squareness errors are the PIGEs for the machine tool. The forward kinematic model considering the geometric errors would be established and the kinematic chains is shown in Fig 2 so that relationship between two adjacent bodies is easily described.

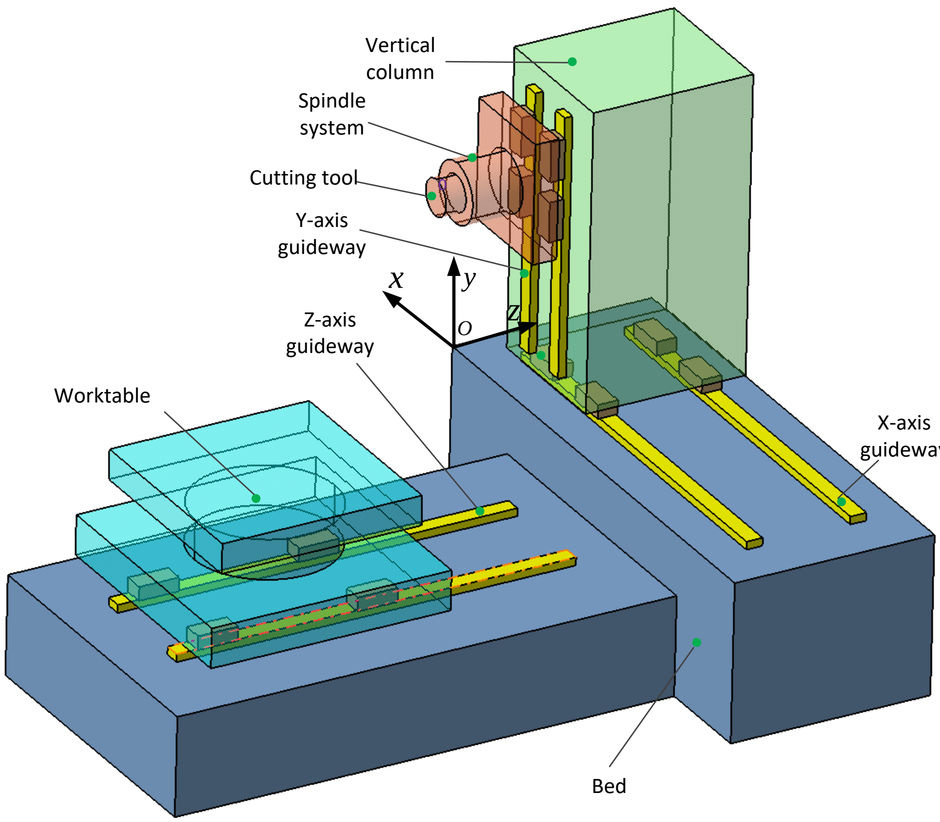


Fig. 1. Composition of the investigated horizontal machine tool

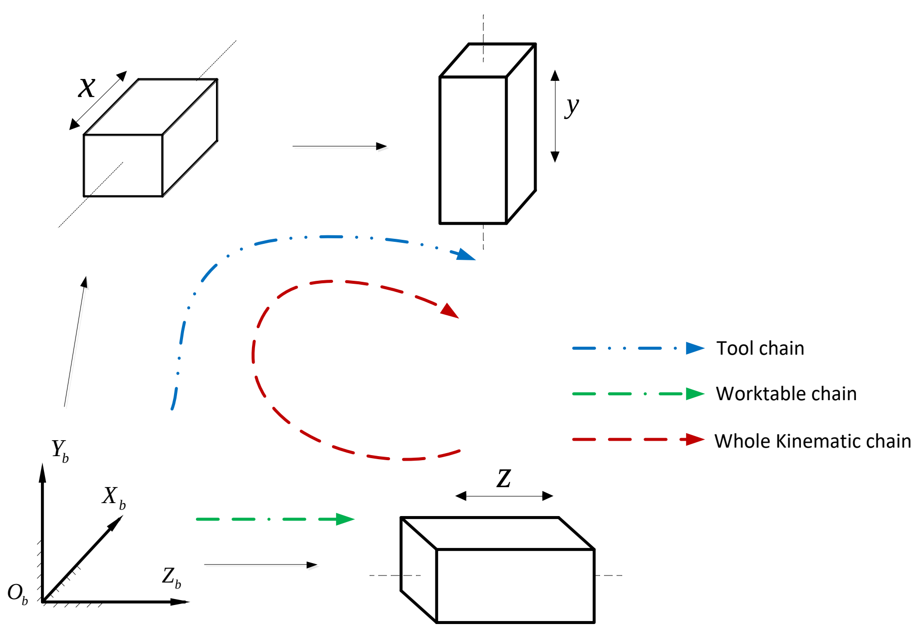


Fig. 2. Kinematic chains of the horizontal machine tool

The whole kinematic chain can be seen as an open kinematic chain consisting of a series of joints with corresponding bodies connected as shown in Fig 2. The open kinematic chain is composed of two separate kinematic chains that are the worktable chain and tool chain. The ground is fixed with base coordinate system, which is seen as the global reference. The worktable moves as Z axis and worktable kinematic chain goes from ground to

Z axis. The tool is fixed to spindle which is connected to the Y axis. The tool kinematic chain goes from ground to X axis, then to Y axis.

All the geometric errors would influence overall volumetric error in the machine tool, then the relationship of the geometric errors and volumetric error can be modeled as presented in the following part.

B. Volumetric Error Modeling

The overall error of the machine tool mainly caused by PDGEs and PIGEs of kinematic joints is seen as volumetric error. The screw theory method is adopted to establish the volumetric errors [2]. The global base coordinate system is the base frame that is fixed on the ground. The tool coordinate frame is a local frame that is on tool tip. The worktable coordinate frame is a local frame that is on center of worktable plane.

The kinematic chains are considered in volumetric error modeling process. The forward kinematics expressions can be presented for the worktable chain and tool chain in the ideal condition that it is with no geometric errors as follows.

$$\begin{cases} g^i_{bw}(z) = e^{\hat{\xi}_Z \cdot Z} \cdot g^i_{bw}(0) \\ g^i_{bt}(x,y) = e^{\hat{\xi}_X \cdot x} e^{\hat{\xi}_Y \cdot y} \cdot g^i_{bt}(0) \end{cases} \quad (1)$$

Where $g^i_{bw}(z)$ and $g^i_{bt}(x,y)$ are Homogeneous Transformation Matrix (HTM) of worktable frame and tool frame relative to base frame in ideal condition. x,y,z are nominal displacements and $\hat{\xi}_X, \hat{\xi}_Y, \hat{\xi}_Z$ are motion twists of corresponding axes. $g^i_{bw}(0)$ and $g^i_{bt}(0)$ are the HTM of the worktable and the tool relative to base frame in reference configuration that is state of the machine tool when all the axes are zero motions.

For whole kinematic chain, forward kinematics expression in the ideal condition is presented as follows.

$$g^i_{wt}(x,y,z) = (g^i_{bw}(z))^{-1} \cdot g^i_{bt}(x,y) \quad (2)$$

Where the notation $g^i_{wt}(x,y,z)$ represents the HTM of tool frame relative to worktable frame in ideal condition.

In actual conditions, the geometric errors would have effects on the forward kinematics. Geometric errors are described by the error motions and error twists. Then the actual forward kinematics expressions of the worktable chain and the tool chain are expressed as follows.

$$\begin{cases} g^a_{bw}(z) = e^{\hat{\xi}_{\Delta Z} \cdot \Delta Z} \cdot e^{\hat{\xi}_Z \cdot Z} \cdot g^i_{bw}(0) \\ g^a_{bt}(x,y) = e^{\hat{\xi}_{\Delta X} \cdot \Delta x} \cdot e^{\hat{\xi}_X \cdot x} \cdot e^{\hat{\xi}_{\Delta y} \cdot \Delta Y} \cdot e^{\hat{\xi}_Y \cdot y} \cdot g^i_{bt}(0) \end{cases} \quad (3)$$

Where the notations $g^a_{bw}(z)$ and $g^a_{bt}(x,y)$ represent the HTMs of worktable frame and tool frame relative to base frame in actual condition.

The error motion screws for X axis, Y axis and Z axis are shown in the followings.

$$\begin{cases} e^{\hat{\xi}_{\Delta X} \cdot \Delta x} = e^{\hat{\xi}_{\delta_{xx}} \cdot \delta_{xx}} \cdot e^{\hat{\xi}_{\delta_{yx}} \cdot \delta_{yx}} \cdot e^{\hat{\xi}_{\delta_{zx}} \cdot \delta_{zx}} \cdot e^{\hat{\xi}_{\varepsilon_{xx}} \cdot \varepsilon_{xx}} \cdot e^{\hat{\xi}_{\varepsilon_{yx}} \cdot \varepsilon_{yx}} \cdot e^{\hat{\xi}_{\varepsilon_{zx}} \cdot \varepsilon_{zx}} \\ e^{\hat{\xi}_{\Delta Y} \cdot \Delta y} = e^{\hat{\xi}_{\delta_{xy}} \cdot \delta_{xy}} \cdot e^{\hat{\xi}_{\delta_{yy}} \cdot \delta_{yy}} \cdot e^{\hat{\xi}_{\delta_{zy}} \cdot \delta_{zy}} \cdot e^{\hat{\xi}_{\varepsilon_{xy}} \cdot \varepsilon_{xy}} \cdot e^{\hat{\xi}_{\varepsilon_{yy}} \cdot \varepsilon_{yy}} \cdot e^{\hat{\xi}_{\varepsilon_{zy}} \cdot \varepsilon_{zy}} \cdot e^{\hat{\xi}_{S_{xy}} \cdot S_{xy}} \\ e^{\hat{\xi}_{\Delta Z} \cdot \Delta z} = e^{\hat{\xi}_{\delta_{xz}} \cdot \delta_{xz}} \cdot e^{\hat{\xi}_{\delta_{yz}} \cdot \delta_{yz}} \cdot e^{\hat{\xi}_{\delta_{zz}} \cdot \delta_{zz}} \cdot e^{\hat{\xi}_{\varepsilon_{xz}} \cdot \varepsilon_{xz}} \cdot e^{\hat{\xi}_{\varepsilon_{yz}} \cdot \varepsilon_{yz}} \cdot e^{\hat{\xi}_{\varepsilon_{zz}} \cdot \varepsilon_{zz}} \cdot e^{\hat{\xi}_{S_{xz}} \cdot S_{xz}} \cdot e^{\hat{\xi}_{S_{yz}} \cdot S_{yz}} \end{cases}$$

(4)

Where the notations $\delta_{ij}(i = x, y\text{and}z, j = X, YandZ)$ and ε_{ij} represent position and angular error for j axis in i direction, respectively. The notations S_{xy} , S_{xz} and S_{yz} represent squareness errors between every two axes correspondingly.

The forward kinematics expression of the whole kinematic chain in actual condition is presented as follows.

$$g^a_{wt}(x,y,z) = (g^a_{bw}(z))^{-1} \cdot g^a_{bt}(x,y) \quad (5)$$

Where the notation $g^a_{wt}(x,y,z)$ represents the HTM of tool frame relative to worktable frame in actual condition.

Then volumetric error is deduced by the deviation of the actual point to the ideal point. The tool point $p_0 = (0,0,0,1)^T$ is set at the origin of the tool frame. Then the volumetric error is expressed as follows.

$$\Delta p = g^a_{wt}(x,y,z) \cdot p_0 - g^i_{wt}(x,y,z) \cdot p_0 \quad (6)$$

The results of the volumetric error can be deduced with the above Eqs. (1-6). The results of volumetric error $\Delta_p = (\Delta e_x, \Delta e_y, \Delta e_z, 0)^T$ is presented as follows.

$$\begin{cases}\Delta e_x = \delta_{xx}+\delta_{xy}-\delta_{xz}+(y_{40}-y-y_{t0})S_{xy}+(z_{50}-z_{t0})S_{xz}+ \\ \quad (z_{t0}-z_{10})\varepsilon_{yx}+(y_{10}-y-y_{t0})\varepsilon_{zx}+(z_{t0}-z_{20})\varepsilon_{yy}+ \\ \quad (y_{20}-y_{t0})\varepsilon_{zy}+(z_{30}+z-z_{t0})\varepsilon_{yz}+(y+y_{t0}-y_{30})\varepsilon_{zz} \\ \Delta e_y = \delta_{yx}+\delta_{yy}-\delta_{yz}+(z_{t0}-z_{60})S_{yz}+(x_{t0}-x_{40})S_{xy}+ \\ \quad (z_{10}-z_{t0})\varepsilon_{xx}+(x_{t0}-x_{10})\varepsilon_{zx}+(z_{20}-z_{t0})\varepsilon_{xy}+ \\ \quad (x_{t0}-x_{20})\varepsilon_{zy}+(z_{t0}-z_{30}-z)\varepsilon_{xz}+(x_{30}-x-x_{t0})\varepsilon_{zz} \\ \Delta e_z = \delta_{zx}+\delta_{zy}-\delta_{zz}+(y_{60}-y-y_{t0})S_{yz}+(x+x_{t0}-x_{50})S_{xz}+ \\ \quad (y-y_{10}+y_{t0})\varepsilon_{xx}+(x_{10}-x_{t0})\varepsilon_{yx}+(y_{t0}-y_{20})\varepsilon_{xy}+ \\ \quad (x_{20}-x_{t0})\varepsilon_{yy}+(y_{30}-y-y_{t0})\varepsilon_{xz}+(x+x_{t0}-x_{30})\varepsilon_{yz}\end{cases}$$

(7)

III. IDENTIFICATION METHOD OF SQUARENESS ERRORS

The squareness errors between two axes are mainly depend on the assembly errors and are not related with the other axis. So when any two linear axes are driven simultaneously with the other linear axis unmoved, the overall displacement errors are only related with PDGEs of each axis and the squareness errors between the two axes, which can also be proved in the following mathematic derivations in this section.

The volumetric error model presented can be applied to calculate overall errors of any point in the workspace of the machine tool. For a face formed by two axes, the face diagonal displacement errors are independent of the other axis. The derivation of identification of squareness error s_{xy} is shown as follows.

The volumetric error of any point in the face are assumed as $e_{1di}(e_{x1i}, e_{y1i}, e_{z1i})$ The unit vector of direction of face diagonal displacement assumed as $u_{XY1}(x_{d1}, y_{d1}, z_{d1})$, the diagonal displacement error can be deduced as

$$\Delta_p l1i = e_{x1i}x_{d1} + e_{y1i}y_{d1} + e_{z1i}z_{d1} \tag{8}$$

Where the notation $i = 1, 2, \ldots, n$, i means the sequence number of measured point and there are n measured points on a face diagonal.

As for the face vector, it has the expression $z_{d1} = 0$ and the driven command of Z axis $z = 0$, so the error e_{z1i}. Then the diagonal displacement error is expressed as

$$\Delta p_{l1i} = e_{x1i}x_{d1} + e_{y1i}y_{d1} \tag{9}$$

When the projected point of investigated point on the face diagonal on the two corresponding axes are not both the measured points, the geometric errors of the projected point on each axis can be obtained by linear interpolation.

The geometric errors of each translational axis is obtained by using the multi-axis calibrator and the face diagonal error is obtained with the aligned laser interferometer. Then the volumetric error parameter in the Eq. (9) is expressed as

$$\begin{cases}e_{x1i} = (y_{40}-y-y_{t0})S_{xy}+\delta_{xx}+\delta_{xy}-\delta_{xz}+(z_{t0}-z_{10})\varepsilon_{yx}+ \\ \quad (y_{10}-y-y_{t0})\varepsilon_{zx}+(z_{t0}-z_{20})\varepsilon_{yy}+(y_{20}-y_{t0})\varepsilon_{zy}+ \\ \quad (z_{30}+z-z_{t0})\varepsilon_{yz}+(y+y_{t0}-y_{30})\varepsilon_{zz}+(z_{50}-z_{t0})S_{xz} \\ e_{y1i} = (x_{t0}-x_{40})S_{xy}+\delta_{yx}+\delta_{yy}-\delta_{yz}+(z_{10}-z_{t0})\varepsilon_{xx}+ \\ \quad (x_{t0}-x_{10})\varepsilon_{zx}+(z_{20}-z_{t0})\varepsilon_{xy}+(x_{t0}-x_{20})\varepsilon_{zy}+ \\ \quad (z_{t0}-z_{30}-z)\varepsilon_{xz}+(x_{30}-x-x_{t0})\varepsilon_{zz}+(z_{t0}-z_{60})S_{yz}\end{cases}$$

(10)

The expressions are complex and they can be reduced by assuming the following expressions,

$$\begin{cases}f_{x1i}(S_{xy}) = (y_{40}-y-y_{t0})S_{xy} \\ V_{x1i} = \delta_{xx}+\delta_{xy}-\delta_{xz}+(z_{t0}-z_{10})\varepsilon_{yx}+ \\ \quad (y_{10}-y-y_{t0})\varepsilon_{zx}+(z_{t0}-z_{20})\varepsilon_{yy}+(y_{20}-y_{t0})\varepsilon_{zy}+ \\ \quad (z_{30}+z-z_{t0})\varepsilon_{yz}+(y+y_{t0}-y_{30})\varepsilon_{zz} \\ C_{x1} = (z_{50}-z_{t0})S_{xz} \\ f_{y1i}(S_{xy}) = (x_{t0}-x_{40})S_{xy} \\ V_{y1i} = \delta_{yx}+\delta_{yy}-\delta_{yz}+(z_{10}-z_{t0})\varepsilon_{xx}+ \\ \quad (x_{t0}-x_{10})\varepsilon_{zx}+(z_{20}-z_{t0})\varepsilon_{xy}+(x_{t0}-x_{20})\varepsilon_{zy}+ \\ \quad (z_{t0}-z_{30}-z)\varepsilon_{xz}+(x_{30}-x-x_{t0})\varepsilon_{zz} \\ C_{y1} = (z_{t0}-z_{60})S_{yz}\end{cases}$$

(11)

It can be seen from the expressions above that $f_{x1i}(S_{xy})$ is the overall error caused by the squareness error at the position numbered i in x direction. V_{x1i} is the overall error caused by all the PDGEs at the position numbered i in x direction. C_{x1} represents the constant value caused by the other two squareness errors in the x direction. $f_{y1i}(S_{xy})$, V_{y1i} and C_{y1} are defined in the same way but in the y direction. So the Eqs. (10) are reduced to the following expressions.

$$\begin{cases} e_{x1i} = f_{x1i}(S_{xy}) + V_{x1i} + C_{x1} \\ e_{y1i} = f_{y1i}(S_{xy}) + V_{y1i} + C_{y1} \end{cases} \tag{12}$$

Then substituting the Eq. (11) in Eq. (12), the above Eq. (12) is expressed as

$$\begin{aligned}\Delta p_{l1i} = [f_{x1i}S_{xy}\cdot x_{d1} + f_{y1i}(S_{xy})\cdot y_{d1}]+ \\ (V_{x1i}\cdot x_{d1} + V_{y1i}\cdot y_{d1}) + (C_{x1}\cdot x_{d1} + C_{y1i}\cdot y_{d1})\end{aligned} \tag{13}$$

The first point of the face diagonal is set as the zero error reference and the actual face diagonal displacement error is expressed as

$$\Delta E_{l1i} = \Delta p_{l1i} - \Delta p_{l11} = \{[f_{x1i}(S_{xy}) - f_{x11}(S_{xy})]\cdot x_{d1} + [f_{y1i}(S_{xy}) - f_{y11}(S_{xy})]\cdot y_{d1}\} + [(V_{x1i} - V_{x11})\cdot x_{d1} + (V_{y1i} - V_{y11})\cdot y_{d1}] = f_{l1i}(S_{xy}) + V_{l1i} \quad (14)$$

In the above Eq. (14), $f_{l1i}(S_{xy})$ is the face diagonal displacement error caused by the squareness error at the position numbered i and V_{l1i} is the face diagonal displacement error caused by all the PDGEs at the position numbered i.

The squareness error S_{xy} is identified by solving the over determined Eqs. (15), $i = 1, 2, \ldots n$.

$$\begin{cases} \Delta E_{l12} = f_{l12}(S_{xy}) + V_{l12} \\ \Delta E_{l13} = f_{l13}(S_{xy}) + V_{l13} \\ \ldots\ldots \\ \Delta E_{l1n} = f_{l1n}(S_{xy}) + V_{l1n} \end{cases} \quad (15)$$

Similarly, the other squareness errors S_{xz} and S_{yz} are also identified in the same method.

IV. EXPERIMENTS AND VERIFICATIONS

According to the identification method of squareness errors presented in Section 3, the measuring and verification of the method is applied in a horizontal machine tool, the composition of which is drawn in Fig 1. Firstly, the PDGEs of the 3 axes are measured using the Renishaw XM60 multi-axis calibrator. The experiment setups of measuring the 3 axes are shown in Fig 3. The ambient temperature is stable and the results are measured more than three times forward and backward. Then the displacement errors of 3 face diagonals are measured using the Renishaw laser interferometer XL80. The setup of each measurement is shown in Fig 4. The 3 face diagonals in the experiments are named as PP-900, P-800P and -500PP. The diagonal PP-900 means the diagonal goes in the positive coordinate commands of X and Y axis at command Z = -900mm. The diagonal P-800P means the diagonal goes in the positive commands of X and Z axis at command Y = -800mm. The diagonal -500PP means the diagonal goes in the positive commands of Y and Z axis at command X = -500mm.

a. Set-up for X axis b. Set-up for Y axis c. Set-up for Z axis

Fig. 3. Set-ups for measuring PDGEs of each axis

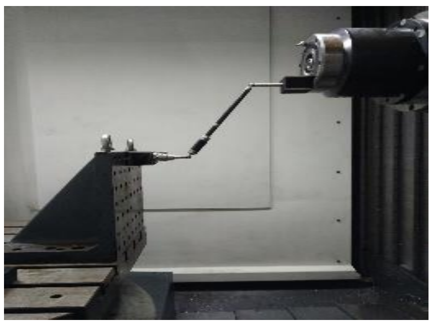
a. PP-900 b. P-800P c. -500PP

Fig. 4. Set-ups of measurement of face diagonal of each plane

Fig 4 Set-ups of measurement of face diagonal of each plane After measuring the PDGEs of each axis and the 3 face diagonal errors, the measured error data is adopted in the identification method presented in Section 3. The data and formulas are programed in the MATLAB software and the identified squareness errors are calculated as

$$S_{xy1} = -30.3\mu m/m, S_{xz1} = -35.9\mu m/m, \\ S_{yz1} = 43.8\mu m/m$$

To verify the squareness errors of the identified value, the measurements of the double ball bar and the mechanical reference square with clock gauge are also implemented in the machine tool (Fig 5). The ball bar is lengthened to 300 mm and calibrated in the measurement to give a more accurate value compared with short ball bar. Each clock gauge value is read carefully and recorded then the fitting line with least square method is adopted to obtain squareness errors.


a. Double ball bar.


b. Mechanical reference square.

Fig. 5. Measurement of squareness errors

The squareness errors measured by the ball bar are shown as

$$S_{xy2} = -28.4\mu m/m, S_{xz2} = -37.4\mu m/m, \\ S_{yz2} = -27.2\mu m/m$$

The squareness errors measured by the mechanical reference square with clock gauge are shown as

$$S_{xy3} = -38.2\mu m/m, S_{xz3} = -15.0\mu m/m, \\ S_{yz3} = -65.5\mu m/m$$

As can be seen from the results, the measured squareness errors are different in some degree by the two different methods. The two measuring methods are popular used in the measurement as presented in ISO 230-1 [32]. Moreover, to give verification and comparisons, the other 3 face diagonals have also been measured, which are named as NP-900, N-800P and -500PN with the similar meaning as clarified previously with other 3 diagonals. Then the results can be used in the diagonal displacement test to obtain the squareness errors as shown in ISO 230-1. The calculated results are shown below. Comparing all the error results with different methods, the identified squareness errors are acceptable in the allowable accuracy range.

$$S_{xy4} = -11.9\mu m/m, S_{xz4} = -12.1\mu m/m, \\ S_{yz4} = -34.4\mu m/m$$

Moreover, the identified squareness errors are also applied in the evaluation of face diagonal errors that can be used in the volumetric performance evaluation as presented in ISO 230-6 [25]. The 6 face diagonal errors of each plane is taken as example. The face diagonal errors are acquired with the error models presented in Section 2 using the identified squareness errors as well as using other measured squareness errors. The face diagonal errors are also measured using the laser interferometer. The plot results are shown in Fig 6.

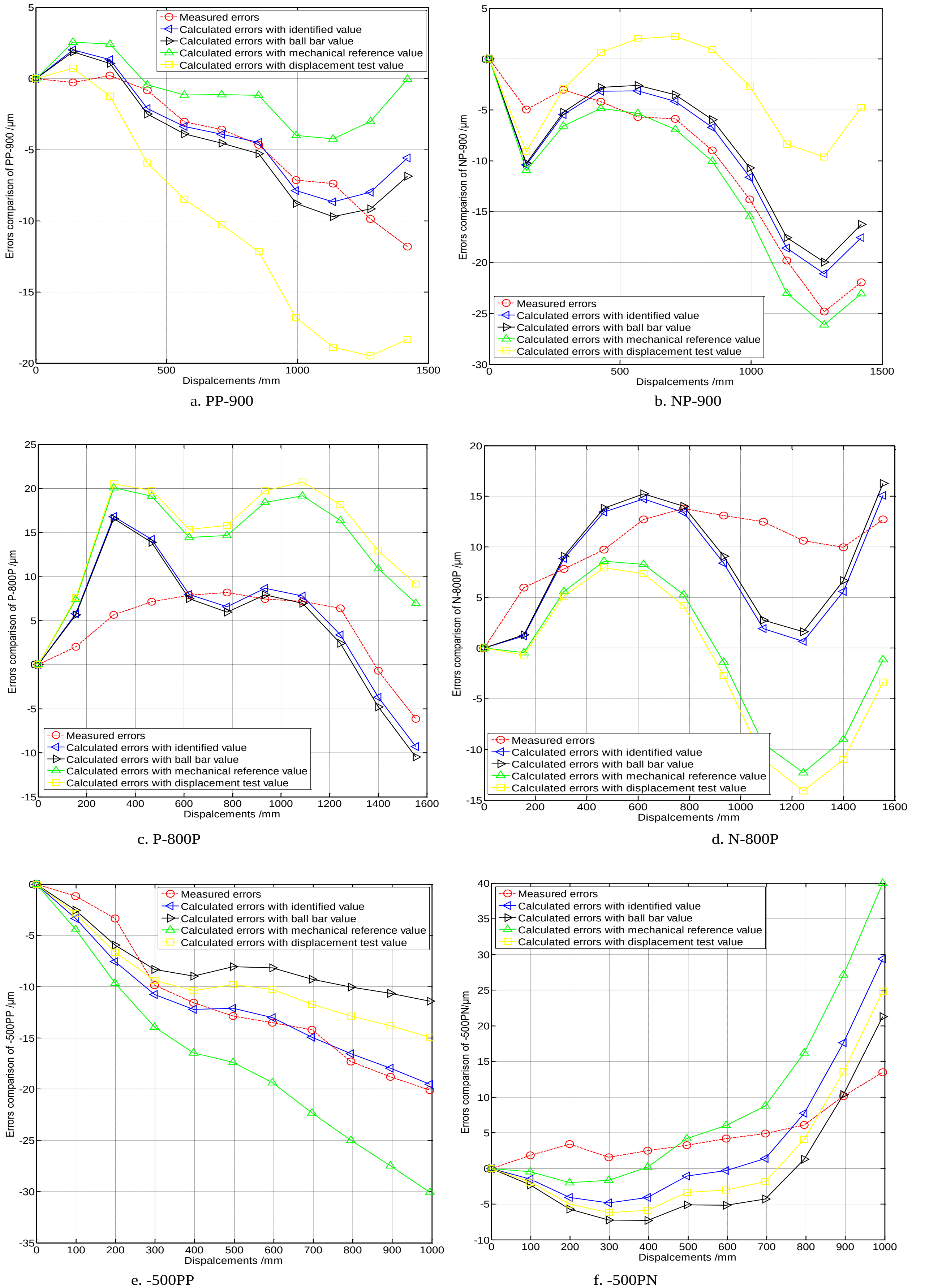


Fig. 6. Comparison of face diagonal errors by measurement and by the calculation with different squareness errors using different methods

It can be seen from Fig 6 that the calculated face diagonal errors with the identified errors accord with the measured diagonal errors in the six subfigures except some errors in a few points compared with other methods. However, other methods using the ball bar test, mechanical reference with clock gauge and diagonal displacement test are not with consistent calculated face diagonal errors in all cases, except some acceptable errors in the figures. The calculated face diagonal errors using the ball bar test value have poor consistency with measured errors with maximum remaining error about 10μm in Fig 6(e) and 6(f). The calculated face diagonal errors using the mechanical reference and clock gauge value have poor consistency with measured errors with maximum remaining error about 25μm in Fig 6(a) and 6(c-f) except in Fig 6(b). The calculated face diagonal errors using the diagonal displacement test value also have poor consistency with measured errors with maximum remaining error about 15μm in Fig 6(a-d). The identified squareness errors show more properly used to predict the corresponding face diagonal errors. It is useful and effective in the volumetric evaluation in some cases, which would also meet commands of overall accuracy enhancement for the machine tools.

V. CONCLUSION AND RECOMMENDATIONS

The overall volumetric errors of the machine tool are investigated to propose a method to measure and identify squareness errors in the paper. The volumetric error model for the volumetric error considering all the geometric errors in a horizontal machine tool is presented using the screw theory. An identification method of squareness errors is presented considering the predicted face diagonal positioning errors. Each squareness error of the corresponding plane that the two axes form and the PDGEs of corresponding axes determine positioning errors of corresponding face diagonals, which is seen in the deduced identification method.

The experiments are implemented in the horizontal machine tool. For each squareness error, the PDGEs of corresponding axes and the positioning errors of one face diagonal are used to implement the identification method. The three squareness errors are identified by the proposed measuring and identification method. Moreover, the squareness errors are also measured by double ball bar test, the mechanical reference square with clock gauge and the diagonal displacement test. The measured squareness errors are different someway by different measuring methods, yet the identified squareness errors are acceptable with the allowable accuracy range compared with the measured results. At last, the face diagonal errors are calculated with the identified squareness errors. The results are compared with the measured face diagonal errors that the calculated errors are in accordance with the measured errors except some errors in a few points. The comparisons of the calculated face diagonal errors by other three methods are also presented. The results show identified squareness errors are more useful and effective in the volumetric evaluation in some special cases, compared with the other three methods with relatively large remaining errors. For the other coupled position dependent geometric error parameters are considered and excluded, this method would be more proper for the volumetric error compensation compared with traditional methods. However, the proposed measuring and identification method shows somewhat complicated and it would cost long time to implement the whole measurement process as can be seen in the experiments. Moreover, the precise measurement by the laser interferometer would be effected by the temperature variation as it costs more time for the temperature to change, which may limit its wide use.

In future study, the measuring and identification method would be researched further combined with volumetric error compensation of the machine tool, as well as the measurement uncertainty in more cases and more application verifications may be proposed.

Declaration of Conflicting Interests

There are no conflicts of interest in this study.

Acknowledgments

This research was supported by the Major Project of China of National Science and Technology under Grant No. 2015ZX04000016, the National Natural Science Fund under Grant No. 51375193 and 51421062.

REFERENCES

[1] S. K. Tan and C. K. Lim, "Modeling the volumetric errors in calibration of five-axis cnc machine," in *International Multiconference of Engineers and Computer Scientists (IMECS 2010),* Hong Kong, China, 2010.

[2] X. Zhong, H. Liu, X. Mao, B. Li, S. He, and F. Peng, "Volumetric error modeling, identification and compensation based on screw theory for a large multi-axis propeller-measuring machine," *Measurement Science and Technology*, vol. 29, no. 5, pp. 1–27, 2018. doi: https://doi.org/10.1088/1361-6501/aaaef3

[3] B. K. Jha and A. Kumar, "Analysis of geometric errors associated with five-axis machining cen-

tre in improving the quality of cam profile," *International Journal of Machine Tools and Manufacture*, vol. 43, no. 6, pp. 629–636, 2003. doi: https://doi.org/10.1016/s0890-6955(02)00268-7

[4] C. J, L. S, and H. B, "Geometric error compensation for multi-axis cnc machines based on differential transformation." *The International Journal of Advanced Manufacturing Technology*, vol. 71, no. 1-4, pp. 635–642, 2014. doi: http://dx.doi.org/10.1007/s00170-013-5487-7

[5] G. Fu, J. Fu, Y. Xu, and Z. Chen, "Product of exponential model for geometric error integration of multi-axis machine tools," *The International Journal of Advanced Manufacturing Technology*, vol. 71, no. 9-12, pp. 1653–1667, 2014. doi: https://doi.org/10.1007/s00170-013-5586-5

[6] J. Yang and Y. Altintas, "Generalized kinematics of five-axis serial machines with non-singular tool path generation," *International Journal of Machine Tools and Manufacture*, vol. 75, pp. 119–132, 2013. doi: https://doi.org/10.1016/j.ijmachtools.2013.09.002

[7] Q. Cheng, Q. Feng, Z. Liu, P. Gu, and G. Zhang, "Sensitivity analysis of machining accuracy of multi-axis machine tool based on poe screw theory and morris method," *The International Journal of Advanced Manufacturing Technology*, vol. 84, no. 9-12, pp. 2301–2318, 2016. doi: https://doi.org/10.1007/s00170-015-7791-x

[8] G. Chen, Y. Liang, Y. Sun, W. Chen, and B. Wang, "Volumetric error modeling and sensitivity analysis for designing a five-axis ultra-precision machine tool," *The International Journal of Advanced Manufacturing Technology*, vol. 68, no. 9-12, pp. 2525–2534, 2013. doi: https://doi.org/10.1007/s00170-013-4874-4

[9] H. Schwenke, W. Knapp, H. Haitjema, A. Weckenmann, R. Schmitt, and F. Delbressine, "Geometric error measurement and compensation of machinesan update," *CIRP Annals*, vol. 57, no. 2, pp. 660–675, 2008. doi: https://doi.org/10.1016/j.cirp.2008.09.008

[10] H. Liu, H. Xiang, J. Chen, and R. Yang, "Measurement and compensation of machine tool geometry error based on abbe principle," *The International Journal of Advanced Manufacturing Technology*, vol. 98, no. 9-12, pp. 2769–2774, 2018. doi: https://doi.org/10.1007/s00170-018-2471-2

[11] M. Vahebi and B. Arezoo, "Accuracy improvement of volumetric error modeling in cnc machine tools," *The International Journal of Advanced Manufacturing Technology*, vol. 95, no. 5-8, pp. 2243–2257, 2018. doi: https://doi.org/10.1007/s00170-017-1294-x

[12] Z. Jiang, B. Song, X. Zhou, X. Tang, and S. Zheng, "On-machine measurement of location errors on five-axis machine tools by machining tests and a laser displacement sensor," *International Journal of Machine Tools and Manufacture*, vol. 95, pp. 1–12, 2015.

[13] S. Xiang, J. Yang, and Y. Zhang, "Using a double ball bar to identify position-independent geometric errors on the rotary axes of five-axis machine tools," *The International Journal of Advanced Manufacturing Technology*, vol. 70, no. 9-12, pp. 2071–2082, 2014. doi: http://dx.doi.org/10.1007/s00170-013-5432-9

[14] J. Li, F. Xie, X. J. Liu, W. Li, and S. Zhu, "Geometric error identification and compensation of linear axes based on a novel 13-line method," *The International Journal of Advanced Manufacturing Technology*, vol. 87, no. 5-8, pp. 2269–2283, 2016. doi: https://doi.org/10.1007/s00170-016-8580-x

[15] S. Ibaraki, C. Oyama, and H. Otsubo, "Construction of an error map of rotary axes on a five-axis machining center by static r-test," *International Journal of Machine Tools and Manufacture*, vol. 51, no. 3, pp. 190–200, 2011. doi: https://doi.org/10.1016/j.ijmachtools.2010.11.011

[16] S. Zargarbashi and J. Mayer, "Assessment of machine tool trunnion axis motion error, using magnetic double ball bar," *International Journal of Machine Tools and Manufacture*, vol. 46, no. 14, pp. 1823–1834, 2006. doi: https://doi.org/10.1016/j.ijmachtools.2005.11.010

[17] A. Lasemi, D. Xue, and P. Gu, "Accurate identification and compensation of geometric errors of 5-axis cnc machine tools using double ball bar," *Measurement Science and Technology*, vol. 27, no. 5, p. 055004, 2016. doi: https://doi.org/10.1088/0957-0233/27/5/055004

[18] W. Lei and Y. Hsu, "Accuracy test of five-axis cnc machine tool with 3d probe–ball. part i: design and modeling," *International Journal of Machine Tools and Manufacture*, vol. 42, no. 10, pp. 1153–1162, 2002. doi: https://doi.org/10.1016/s0890-6955(02)00048-2

[19] M. Jankowski and A. Wozniak, "Mechanical model of errors of probes for numerical controlled machine tools," *Measurement*, vol. 77, pp. 317–326, 2016. doi: https://doi.org/10.1016/j.measurement.2015.09.023

[20] S. Aguado, J. Santolaria, J. Aguilar, D. Samper,

and J. Velazquez, "Improving the accuracy of a machine tool with three linear axes using a laser tracker as measurement system," *Procedia engineering*, vol. 132, pp. 756–763, 2015. doi: https://doi.org/10.1016/j.proeng.2015.12.557

[21] S. Aguado, D. Samper, J. Santolaria, and J. J. Aguilar, "Identification strategy of error parameter in volumetric error compensation of machine tool based on laser tracker measurements," *International Journal of Machine Tools and Manufacture*, vol. 53, no. 1, pp. 160–169, 2012. doi: https://doi.org/10.1016/j.ijmachtools.2011.11.004

[22] Z. Zhang and H. Hu, "Measurement and compensation of geometric errors of three-axis machine tool by using laser tracker based on a sequential multilateration scheme," *Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture*, vol. 228, no. 8, pp. 819–831, 2014. doi: https://doi.org/10.1177/0954405413511607

[23] A. H. Jalaludin, M. H. A. Shukor, N. A. Mardi, A. A. D. M. Sarhan, M. S. Ab Karim, S. R. Besharati, W. N. I. W. Badiuzaman, and Y. S. Dambatta, "Development and evaluation of the machining performance of a cnc gantry double motion machine tool in different modes," *The International Journal of Advanced Manufacturing Technology*, vol. 93, no. 1-4, pp. 1347–1356, 2017. doi: https://doi.org/10.1007/s00170-017-0332-z

[24] D.-M. Lee, H.-H. Lee, and S.-H. Yang, "Analysis of squareness measurement using a laser interferometer system," *International Journal of Precision Engineering and Manufacturing*, vol. 14, no. 10, pp. 1839–1846, 2013. doi: https://doi.org/10.1007/s12541-013-0246-0

[25] International Organization for Standardization Patent ISO 230-6, Test code for machine tools Part 6:Determination of positioning accuracy on body and face diagonals (Diagonal displacement tests), 2002.

[26] J.-P. Kruth, L. Zhou, C. Van Den Bergh, and P. Vanherck, "A method for squareness error verification on a coordinate measuring machine," *The International Journal of Advanced Manufacturing Technology*, vol. 21, no. 10-11, pp. 874–878, 2003. doi: https://doi.org/10.1007/s00170-002-1408-x

[27] M. Rahman, J. Heikkala, and K. Lappalainen, "Modeling, measurement and error compensation of multi-axis machine tools. part i: theory," *International Journal of Machine Tools and Manufacture*, vol. 40, no. 10, pp. 1535–1546, 2000. doi: https://doi.org/10.1016/s0890-6955(99)00101-7

[28] G. Fu, J. Fu, H. Gao, and X. Yao, "Squareness error modeling for multi-axis machine tools via synthesizing the motion of the axes," *The International Journal of Advanced Manufacturing Technology*, vol. 89, no. 9-12, pp. 2993–3008, 2017. doi: https://doi.org/10.1007/s00170-016-9259-z

[29] Y. Liu, H. Zhang, and X. Wang, "Analysis on influence of perpendicularity error of five axis nc machine tool error modeling accuracy and complexity," *Procedia engineering*, vol. 174, pp. 557–565, 2017. doi: https://doi.org/10.1016/j.proeng.2017.01.187

[30] S. Xiang and Y. Altintas, "Modeling and compensation of volumetric errors for five-axis machine tools," *International Journal of Machine Tools and Manufacture*, vol. 101, pp. 65–78, 2016.

[31] J. Creamer, P. M. Sammons, D. A. Bristow, R. G. Landers, P. L. Freeman, and S. J. Easley, "Table-based volumetric error compensation of large five-axis machine tools," *Journal of Manufacturing Science and Engineering*, vol. 139, no. 2, p. 021011, 2017. doi: https://doi.org/10.1115/1.4034399

[32] International Organization for Standardization Patent ISO 230-1:2012: Test code for machine tools – Part 1: Geometric accuracy of machines operating under no-load or quasi-static conditions, 2012.