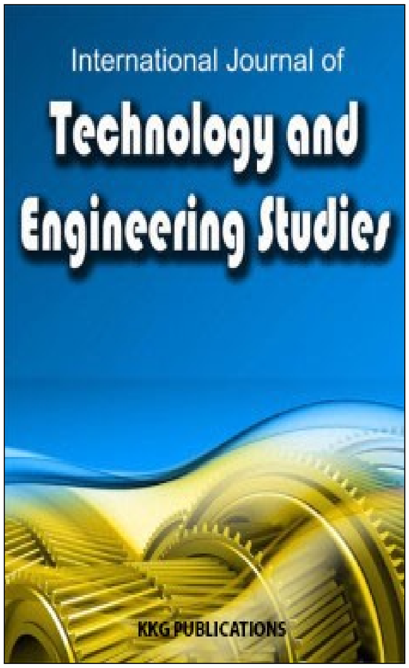

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Published online: 17 August 2017

To cite this article: O. Barkat and B. Mamri, "The effect of incident angle and filling factor on dispersion properties of different structures of one-dimensional photonic crystals," *International Journal of Technology and Engineering Studies,* vol. 3, no. 4, pp. 133-140, 2017.
DOI: https://dx.doi.org/10.20469/ijtes.3.40001-4

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International Journal of Technology and Engineering Studies
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IJTES

THE EFFECT OF INCIDENT ANGLE AND FILLING FACTOR ON DISPERSION PROPERTIES OF DIFFERENT STRUCTURES OF ONE-DIMENSIONAL PHOTONIC CRYSTALS

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Keywords:
One-Dimensional Photonic Crystal
Transfer Matrix Method
Bloch Theorem
Dispersion

Abstract. This study investigates the effect of filling factor and incident angle on the dispersion curve of (1D-PC) structures. This paper applied the transfer matrix method in conjunction with the Bloch theorem to study the dispersion properties of one-dimensional photonic crystals made by TiO_2/ GaAs and TiO_2/ MgF_2. The effects of filling factor and incident angle on the photonic gap map are investigated for both TE and TM polarizations. It is found that the location, number, and width of the photonic gap are affected significantly by the filling factor and incident angle of the structure. Results are compared with published data and are found to be in good agreement. These approaches can allow for better characterization of multilayer metamaterial structures.

Received: 15 April 2017
Accepted: 22 June 2017
Published: 17 August 2017

INTRODUCTION

The One-Dimensional Photonic Crystal (1D-PC) structures have become attractive to optical engineering due to their several beneficial features, such as the ability to control and manipulate the propagation of electromagnetic waves in limited space. These structures have found use in many different applications since their discovery, due to their excellent advantages [1], [2], [3]. These structures have a number of useful properties, which are employed as lenses, mirrors, optical buffer, optical multiplexers, optical filters, etc. It has been proved experimentally and theoretically that one-dimensional photonic crystals have complete omnidirectional Photonic Band Gaps (PBGs).

Therefore, several techniques have been proposed for calculation of the dispersion properties of one-dimensional photonic crystals. These methods contain the Plane Wave Method (PWM), Finite Element Method (FEM), Finite-Difference Time Domain Method (FDTD), Finite-Difference Frequency Domain Method (FDFD), Finite Integration Technique (FIT), and the Transfer Matrix Method (TMM). Each method has its own limitations for finding the band structure [4], [5], [6]. Each method has its own limitations for finding the band structure. The transfer matrix method has been developed over the years, and has been widely used to model complex structures. The characteristics of electromagnetic wave transmission behavior of PBG materials were first studied by Pendry and MacKinnon using a matrix representation of each layer TMM. In this paper, a combination of TMM and Block theorem is applied to investigate the effect of filling factor and incident angle on dispersion curve of (1D-PC) structures.

LITERATURE REVIEW

Over the past decades, much attention has been given to photonic crystals (1D-PC). These have been focused on material research of lenses, mirrors, optical buffers, optical multiplexers, and optical filters. [7] first experimented with periodic multi-layer dielectric stacks, showing that such systems have a one-dimensional photonic band-gap, a spectral range of large reflectivity. [8] gave a firm theoretical treatment of 1D photonic band gap structures. However, many ideas did not take off until after the publication of two milestone papers in [9] and [10]. After 1987, the number of research papers concerning photonic crystals began to grow exponentially. In [11] reported for the first time that one-dimensional dielectric lattice displays total omni-directional reflection for incident light under certain conditions. Also [12] demonstrated that one-dimensional photonic crystal structures such as multilayer films can exhibit complete reflection of radiation in a given frequency range for all incident angles and polarizations. [13] showed theoretically that the omnidirectional total reflection frequency range of a multilayer dielectric reflector can be substantially enlarged as desired by using photonic heterostructures [13], [14]. Various materials have been employed in order to obtain one-dimensional photonic crystals (1D) as: Si, SiO_2,

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TiO_2, MgF_2, ZnSe, Ge, and GaAs [4], [15], [16]. In the current paper, we have used TiO_2/ GaAs and TiO_2/ MgF_2 that are three of the preferred materials for building one-dimensional photonic crystals because these present very different dielectric constants. The normalized frequency versus the incident angle was obtained by applying the transfer matrix formalism and Bloch theorem. Several simulation cases by Matlab will be given to show the performance of this approach. The accuracy of the analysis is tested by comparing the computed results with measurements published data.

MATHEMATICAL FORMULATION

Figure 1 shows the configuration of the proposed structure. The (1D-PC) structure is composed of an alternating multilayer having N layer or (N/2) period made up of dielectric materials, every layer having thicknesses dl, and index nl.

The relative permittivity of considered structure can be given as;

$$\varepsilon_l = \begin{cases} \varepsilon_1 & 0 < z < d_1 \\ \varepsilon_2 & d_1 < z < d_2 \end{cases} \tag{1}$$

and

$$\varepsilon_l(z) = \varepsilon_l(z + d) \tag{2}$$

where l: is number of layer
$d = d_1 + d_2$: is period

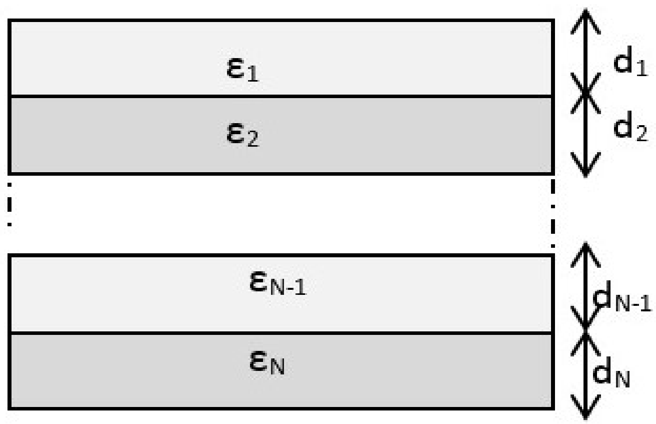

Fig. 1. Geometry of one-dimensional photonic crystal

The analysis necessary to obtain the dispersion properties of (1D-PC) structure needs to calculate the relationship between the fields (1D-PC) structure consisting of l layer. Based on the Maxwell equations and the boundary conditions, the Transfer Matrix (TM) method has been widely used to calculate the amplitude and phase spectra of the light wave propagating in a (1D-PC) structure. In this approach was considered oblique incidence of the incident light. By using the boundary conditions and the condition of continuity of *E* and *H* fields at the interfaces of z=*0* and $z = d_1, d_2, d_3, \ldots d_N$ we can find the relationship between the fields (1D-PC) structure consisting of l layer. This relation is given by [12]:

$$\begin{bmatrix} E_1 \\ H_1 \end{bmatrix} = M_1 M_2 \ldots\ldots M_k \ldots M_{l-1} M_l \begin{bmatrix} E_l \\ H_l \end{bmatrix} \tag{3}$$

The matrix M_{l-1} of the l^{th} layer can be written in the form [17], [18].

$$M_{(l-1)} = \begin{bmatrix} cos(\delta_{(l-1)}) & i\gamma_{(l-1)} sin(\delta_{(l-1)}) \\ i{\gamma_{(l-1)}}^{-1} sin(\delta_{(l-1)}) & cos(\delta_{(l-1)}) \end{bmatrix} \tag{4}$$

Where $\delta_{(l-1)}$ and $\gamma_{(l-1)}$ being the matrix parameters and depending on the incident angle of light, the optical constants and the layer thickness are expressed as:

$$\gamma_{(l-1)} = k_{(l-1)}.d_{(l-1)}.cos\theta_{(l-1)} \tag{5}$$

and

$$\gamma_{(l-1)} = \begin{cases} \frac{\eta_{(l-1)}}{cos\theta_{(l-1)}} & \text{TE mode} \\ \eta_{(l-1)} cos\theta_{(l-1)} & \text{TM mode} \end{cases} \tag{6}$$

We note that $\theta_{(l-1)}$ is related to the angle of incidence θ_0 by the Snell's Descart's law, that is

$$n_{(l-1)} sin\theta_{(l-1)} = n_0 sin\theta_0 \tag{7}$$

In general, wave propagation in periodic media can be described in terms of Bloch waves. For determination of the dispersion surfaces of a periodic crystal, it is necessary only to integrate the wave-field through a periodic media.

According to Bloch theorem, fields in a periodic structure satisfy the following equations [19], [20], [21]:

$$E(z+d) = e^{ikd}E(z) \tag{8}$$

The parameter *k* is called the Bloch wave number. In order to determine *k*, we can use relation between the electric field amplitudes of two layers. From equation (3), we obtain:

$$\begin{bmatrix} E_1 \\ H_1 \end{bmatrix} = M_1 M_2 \begin{bmatrix} E_2 \\ H_2 \end{bmatrix} \tag{9}$$

We can put the product matrix as:

$$M_1.M_2 = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \tag{10}$$

$Tr[M_1.M_2]$ is the trace of the transfer matrix characterizing the wave scattering in a periodic structure, as given by;

$$Tr[M_1.M_2] = M_{11} + M_{22} = 2cos(kd) \tag{11}$$

Where

$$M_{11} = cos(\delta_1) * cos(\delta_2) - (\gamma_1/\gamma_2)\, sin(\delta_1)\, sin(\delta_2) \tag{12}$$

$$M_{21} = cos(\delta_1) * cos(\delta_2) - (\gamma_2/\gamma_1)\, sin(\delta_1)\, sin(\delta_2) \tag{13}$$

Substituting (12) and (13) into (11), we obtain the following equation [12]:

$$cos(kd) = cos(\delta_1) * cos(\delta_2) - (\frac{\gamma_2^2 + \gamma_1^2}{2\gamma_1\gamma_2})\, sin(\delta_1)\, sin(\delta_2) \tag{14}$$

The quantity cos(kd) determines the band structures of the (1D-PC) structure.

RESULTS AND DISCUSSION

In this section, we consider oblique incidence of the electromagnetic wave on the (1D-PC) structure. The numerical tools used for our simulations are based on the TMM and Bloch theorem. The gap map shown in Figures 2-7 represent results in terms of frequencies ($\frac{fd}{c}$) (where *d* is the period constant, *C* the speed of light, and *f* the frequency) as function of the incidence angle (θ) and filling factor ($f = d_1/d$). The red region indicates the variation of TE PBG, the blue region indicates the variation of TM PBG, and the empty space regions represent the ranges of transmission. As a validation of the different results, we have compared our simulation results with the proposed model of [18]. Numerical evaluations have been done with a (1D-PC) structure of optical contrast $n_L/n_H = 3.42/1$ and filling factor $f = 0.774$, shown in Figure 2 and Figure 3. The codes are implemented in Matlab programming software. Our results agree very well with published data obtained by [22].

Shown in Figures 2 and Figure 3 is the dependence of incident angle and filling factor on the photonic band gap width. It is observed that the disappearance of photonic band gaps in TM mode occurs at certain angles of incidence. This effect was explained by the small decrease in the reflection coefficient in the photonic band gap region.

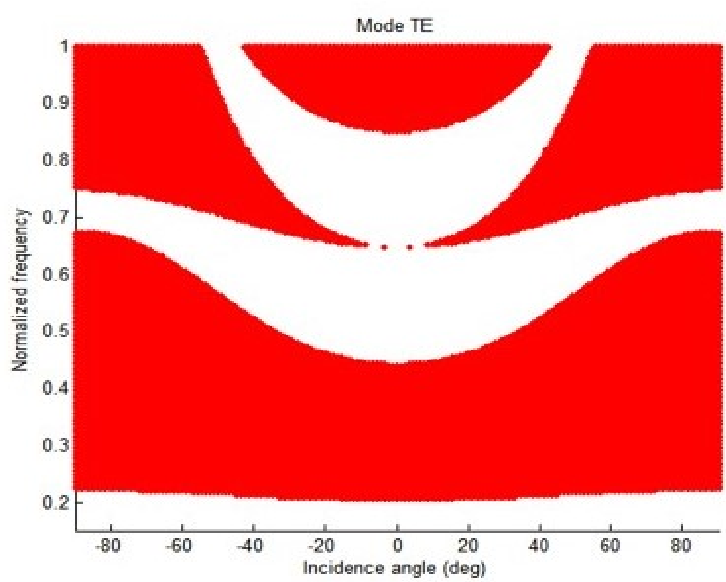

Fig. 2. Normalized frequency versus the incident angle, $n_H = 1$, $n_L = 3.42$, $d_1 = 0.774\mu m$, Mode TE.

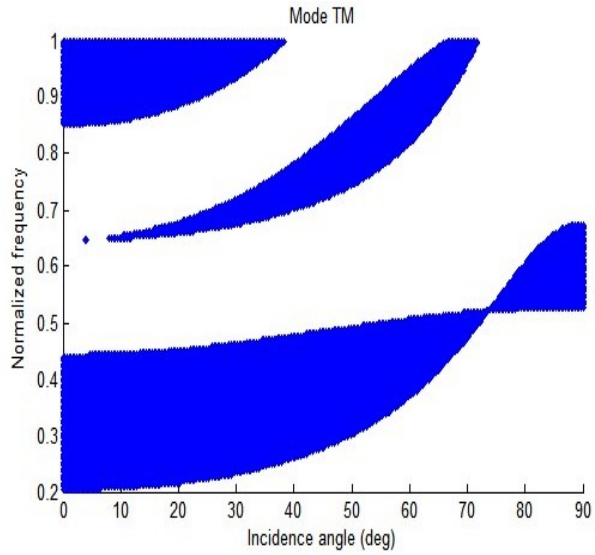

Fig. 3. Normalized frequency versus the incident angle, $n_H = 1$, $n_L = 3.42$, $d_1 = 0.774\mu m$, Mode TM.

Shown in Figures 4-7 is the normalized frequency versus the incident angle (θ) for (*TM*,*TE*) representation. In Figures 4-5 is the (1D-PC) structure of optical contrast $n_L/nH = 1.3743$, containing Titanium dioxide (TiO_2) and Gallium Arsenide (GaAs). We have kept constant the dielectric permittivity's of the layers, are taken as n_H = $n_{\mathrm{TiO}_2} = 2.4538$ and $n_L = n_{GaAs} = 3.3723$ respectively at $\lambda = 1.53\mu m$. The thicknesses of the considered layers that we will use in our studies are: $(d_{\mathrm{TiO}_2} = 0.5788\mu m, d_{GaAs} = 0.4212\mu m)$. In Figures 6-7, we have plotted the incident angle -dependent normalized frequency for (1D-PC) structure of optical contrast $n_L/n_H = 0.5587$, containing Titanium dioxide (TiO_2) and Magnesium fluoride (MgF_2). We have kept constant the dielectric permittivity of the layers that are taken as $n_H = n_{\mathrm{TiO}_2} = 2.4538$ and as n_L = $n_{\mathrm{MgF}_2} = 1.3710$ respectively at $\lambda = 1.53\mu m$. The layers' thicknesses were taken as $d_{\mathrm{TiO}_2} = 0.3585\mu m, d_{\mathrm{MgF}_2} = 0.6415\mu m$.

As seen and extracted from the Figures. 4-7, we have demonstrated that it is possible to modify the PBG of structure by varying the incident angle.

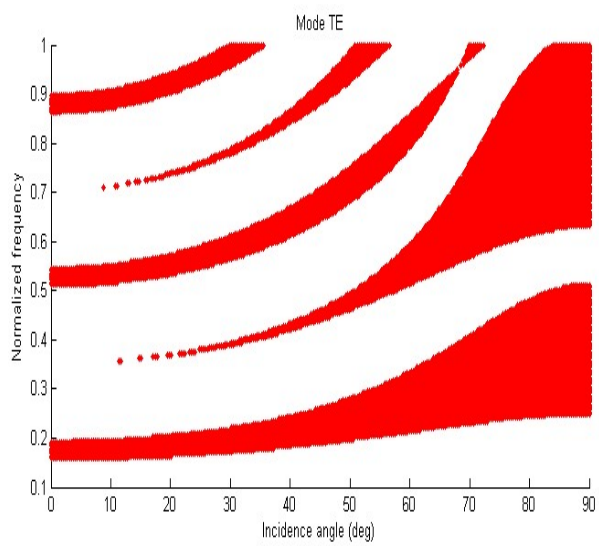

Fig. 4. Normalized frequency versus the incident angle, $n_{GaAs} = 3.3723$, $n_{\mathrm{TiO}_2} = 2.4538$, $d_{GaAs} = 0.4212\mu m$, and $d_{\mathrm{TiO}_2} = 0.5788\mu m$, Mode *TE*.

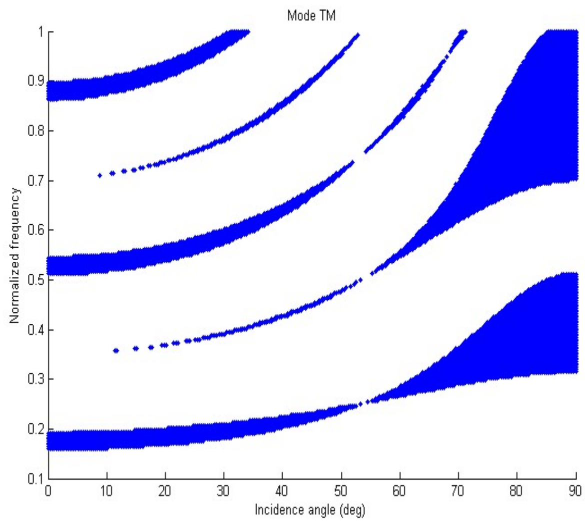

Fig. 5. Normalized frequency versus the incident angle, $n_{\mathrm{TiO}_2} = 2.4538$, $n_{GaAs} = 3.3723, d_{GaAs} = 0.4212\mu m$, and $d_{\mathrm{TiO}_2} = 0.5788\mu m$, Mode *TM*.

It may be seen from the plot that the forbidden bandwidth varies with the incident angle of the two modes. As the angle of the incidence increases, the forbidden bandwidth was also found to be increased and shifted to higher frequency regions. From simulation analysis, we observed that the first band gap appears when the value of θ is greater than 0.001. On the other hand, we can see that our structures exhibit a large band gap for less optical contrast.

Numerical results for the effect of the the ratio filling factor (d_1/d) for oblique incidence ($\theta = 45^o$) on the normalized frequency are shown in Figures 8-11. It may be seen from the plot that the photonic band gap width varies with the ratio of filling factor. For the great values of frequencies, there is discontinuity in photonic band gaps at certain filling factor. In addition, the high value of photonic band gap width for *TE* modes is observed than those of *TM* modes at all frequencies. As we have said previously, a decrease in the width of the band gap was obtained for the *TM* and *TE* modes for small values of optical contrast.

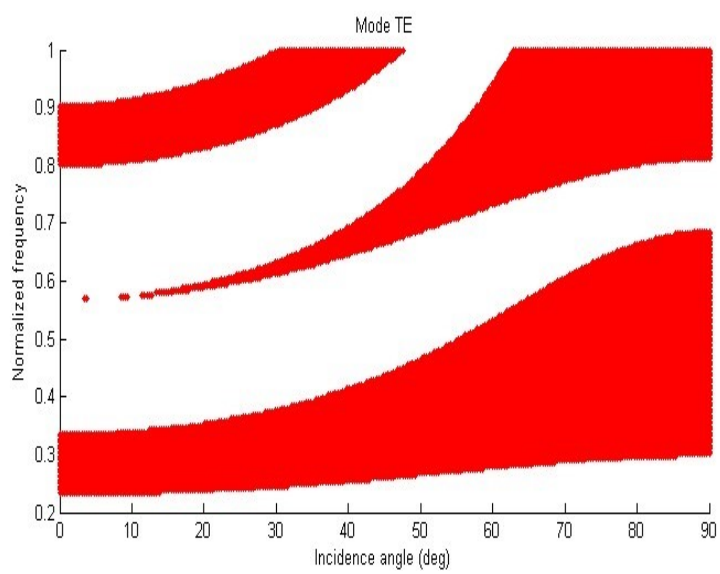

Fig. 6. Normalized frequency versus the incident angle (θ) $n_{\mathrm{TiO}_2} = 2.4538, n_{\mathrm{MgF}_2} = 1.3710$, $d_{\mathrm{TiO}_2} = 0.3585\mu m$, and $d_{\mathrm{MgF}_2} = 0.6415\mu m$, Mode *TE*.

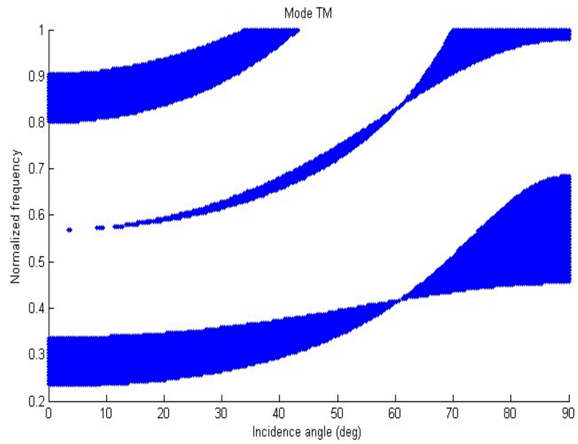

Fig. 7. Normalized frequency versus the incident angle (θ) $n_{\mathrm{TiO_2}} = 2.4538, n_{\mathrm{MgF_2}} = 1.3710$, $d_{\mathrm{TiO_2}} = 0.3585\mu m$, and $d_{\mathrm{MgF_2}} = 0.6415\mu m$, Mode *TM*.

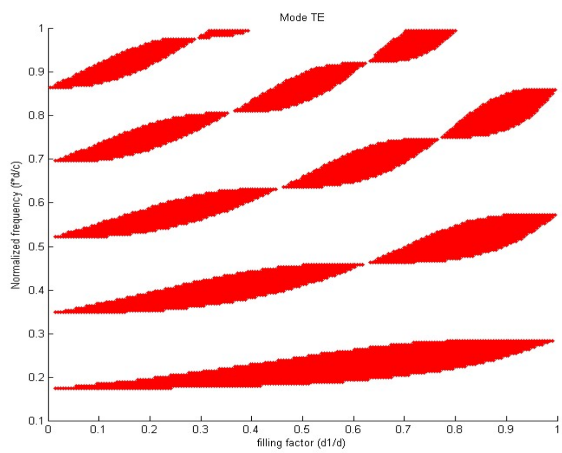

Fig. 8. Normalized frequency versus the filling factor, $n_{\mathrm{TiO_2}} = 2.453, n_{GaAs} = 3.3723$, Mode *TE* $\theta = 45^o$.

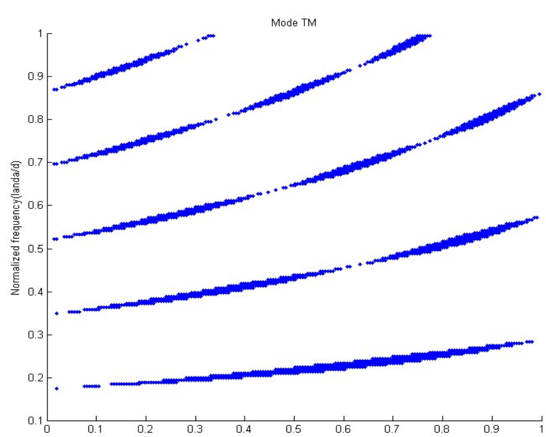

Fig. 9. Normalized frequency versus the filling factor, $n_{\mathrm{TiO_2}} = 2.453, n_{GaAs} = 3.3723$, Mode *TM* $\theta = 45^o$.

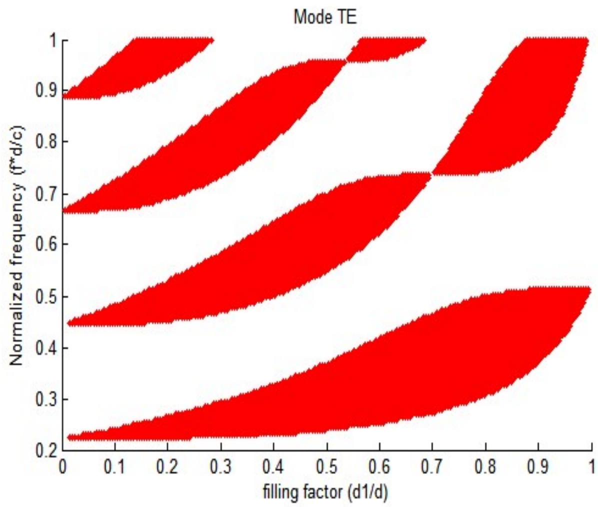

Fig. 10. Normalized frequency versus the filling factor, $n_{\mathrm{TiO_2}} = 2.453, n_{\mathrm{MgF_2}} = 1.3710$, Mode *TE* $\theta = 45^o$.

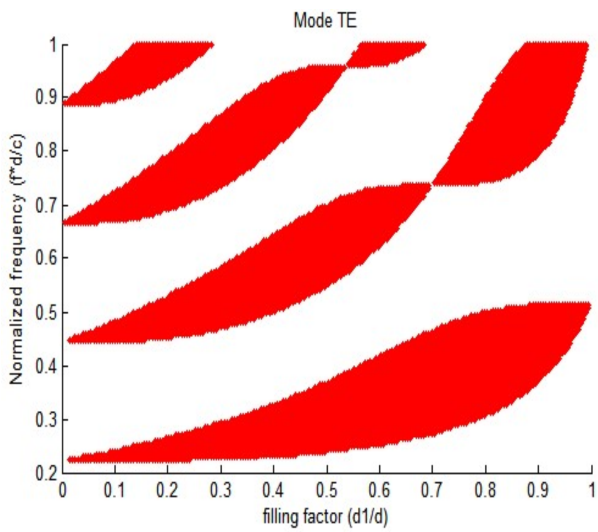

Fig. 11. Normalized frequency versus the filling factor, $n_{\mathrm{TiO_2}} = 2.453, n_{\mathrm{MgF_2}} = 1.3710$, Mode *TM* $\theta = 45^o$.

CONCLUSION AND RECOMMENDATIONS

In this work, we have described an accurate analysis of dispersion properties of 1D-PC. The method is based on calculating the relationship between the fields (1D-PC) structure consisting of l layer and Bloch theorem. Also, we have investigated the dispersion properties for 1D photonic structure made by TiO_2/ GaAs and TiO_2/ MgF_2. Our results show that the incidence angle has an important influence on the photonic band map for both TE and TM modes. The filling factor is also another factor which affects the width of band gaps. Further, these approaches can allow for better characterization of multi-layer metamaterial structures.

Declaration of Conflicting Interests

No known conflicts of interest are declared for the present work.

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