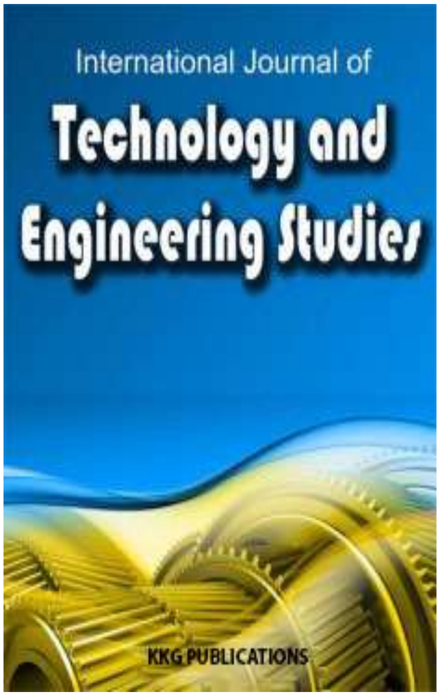

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NUMERICAL STUDY ON OPTIMIZATION OF WOODEN-STEEL HYBRID BEAMS BASED ON SHAPE FACTOR OF STEEL COMPONENT

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Abstract. The aim of this study is to follow and develop a method in order to provide more efficient shape factor performance for Flitch-beams-the first generation of timber-steel-hybrid beams. Basing on the main concept of data tables from the National Association of Home Builders of the United States (NAHB) builders' beam showing the capacity of Flitch-beams with a variety of sizes, this study chooses one fixed pattern of 2 wood pieces (2x8'') inserted by a straight steel-plate-core to evaluate and determine the optimized morphology among the variety of cross-sections. It is then applied into the beam as a replacement for straight steel-plate-core. Besides that, to figure out the optimal core, the three types of Rectangular-section, Hollow-section, and I-section based on the same area condition and material should be assessed properly. The first result showed that the comparison relies on numerical data of Maximum bending stress and deflection at mid-span. Following this, the new Flitch-beam was re-calculated, and the experiment was conducted compared to the NAHB builders' beam pattern. Similarly, the final result indicated that the coordination of the optimized steel core is more advantageous than the patterned beam.

INTRODUCTION

In order to pursue sustainable energy, conserving energy and reducing CO_2 emissions should be the leading tasks. Most of CO_2 emission in the world is produced from the Building Construction, there is 47% CO_2 emission coming from Buildings while the figures for Transportation and Industry are 33% and 19% respectively [1]. Concrete production contributes 5 per cent to annual anthropogenic global CO_2 production [2]. A recent study [3] shows that each 136 m^2 floor of Wooden construction discharges 8,331kg of CO_2, and building with a Wooden structure can give off a fixed amount of CO_2 i.e. 26,524kg (equivalent sequestration of 17,227kg). For those reasons, steel and timber hybridization are less common but they still potentially exist.

Despite of the incompatibility between the different material properties, the light, cheap, and environment-friendly nature of wood makes it a good material to pair with something strong and ductile [4]. In this research topic several different cross-sectional alternatives were given and compared. The three different types of factors are Rectangular-section, Hollow-section and I-section which are primarily shown by Moment of inertia value *(I)* between different sections. The numerical study is to analyze stress in different cross-sections because Elastic theory is mainly determined by cross-sectional beams. The acquisition of factors was defined and thus optimal cross section morphology was determined.

In the future research, the optimal cross section can be applied into the hybridization called "Flitch- beams" to develop a method that provides a searchable manual, in which the higher efficiency of the single timber-steel-hybrid beams is suggested and can help engineers solve the sub-problem of optimizing single beams.

LITERATURE REVIEW

Buildings in Japan have been constructed by using timber since olden times. The five-storey Kanazawa M building was constructed in 2004 in Japan [5]. The columns, beams and braces of the building consist of timber with built-in steel materials. The cross section of each member is shown in Fig. 1 which reports the structural and fire resistance capability.

The idea is to optimize the cross section of the timber-steel hybrid beam and the steel is to be placed inside the wood (Fig. 2), in which fire is prevented and both materials, under a common deformation, can reach at the same time its performance limit value [6].

There is also a design optimum cross-section for such load-carrying members. That involves using a multi-objective evolutionary algorithm for simultaneously maximizing moment of inertias and minimizing the cross-sectional areas that has been carried out by [8], [7], [11]. Another study choosing several different cross-section alternatives with comparison via a

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mathematical nonlinear optimization model is performed by [7]. After the optimization is done and optimal measurements for different cross-sections are found, the results are verified by using finite element method [8].

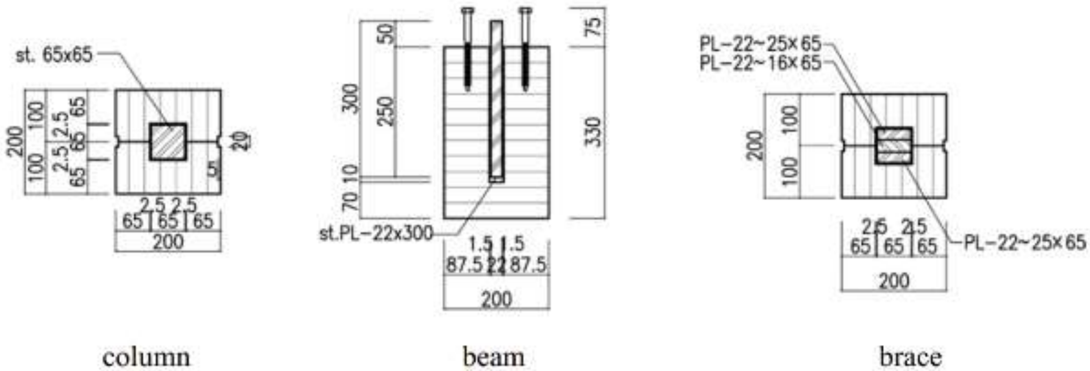


Fig. 1. Cross sections of column, beam, and brace of Kanazawa M building.

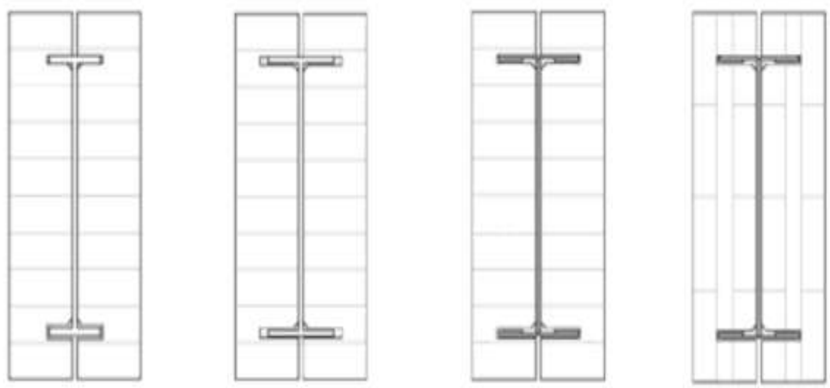


Fig. 2. Cross section of hybrid beams tested, Vienna, Austria.

RESEARCH MODEL

Numerical Methodology of Shape Factor

Relevant Formula and Definitions Review

Moment of inertia (I)

The second moment of area, also known as moment of inertia of plane area is a geometrical property of an area which reflects how its points are distributed with regards to an arbitrary axis. The second moment of area is typically denoted with either an I_x for an axis that lies in the plane or an I_y for an axis perpendicular to the plane. Its unit of dimension is length to fourth power, L^4. Moment of inertia value is calculated by the formula: $I_x = \frac{bh^3}{12}$ (E1).

Section Modulus (S)

Section modulus is a geometric property for a given cross-section which is used in the design of beams or flexural members. Other geometric properties used in design include area for tension, radius of gyration for compression, and moment of inertia for stiffness. Any relationship between these properties highly depends on the shape in question. Equations for the section moduli of common shapes are given as follows:

$$S = S_t = S_b = \frac{bh^3/12}{\frac{1}{2}h} \quad \text{(E2)}$$

Maximum Bending Stress at Mid Span (σ) and Maximum Deflection at Mid Span ($\frac{\Delta}{l}$)

In the field of structural engineering, the second moment of area of the cross-section of a beam is an important property which is used in the calculation of the beam's deflection and the calculation of stress caused by a moment applied to the beam.

- The maximum bending stress at mid span: $\sigma = \frac{M}{S} = \frac{ql^2}{8S} \le f_{t,k}$ (E3) where $f_{t,k}$ is allowable bending stress of timber

- The maximum deflection at mid span: $\frac{\Delta}{l} = \frac{5}{384} \times \frac{ql^3}{EI} \le \left[\frac{\Delta}{l}\right]$

(E4) where $\left[\frac{\Delta}{l}\right]$ is allowable unit deflection

From (E3), (E4) the allowable span of section with moment M is identified by 2 compulsory conditions given characterized by two quantities S (E2) and I (E1) value, the maximum length of the beams is determined by the combination of (E*3) & (E*4) as the following formula:

$$l \le \min\left(\sqrt{\frac{f_{t,k} \times 8S}{q}}, \sqrt[3]{\frac{384}{5} \times \frac{[\Delta / l]EI}{q}}\right) \quad \text{(E5)}$$

If we have 2 sections (I_1 , S_1) (I_2 , S_2), the ratio between maximum span l_1, l_2 is:

$$\frac{l_1}{l_2} \le \min\left(\sqrt{\frac{S_1}{S_2}}, \sqrt[3]{\frac{I_1}{I_2}}\right) \text{(E6).}$$

This equation gives the idea of how many times of the length span the modified beam can reach over the original beam.

Shape Factors

Three different cross-section shapes are chosen for calculation (*Figure 3)* – including: original form: Rectangular-Section, simple form: Hollow-Section and common shape: I-Section. It is found that there is incremental complexity since the dimensions of the beam are modified at the width (*b*) and height (*h*), therefore, these shapes were chosen.

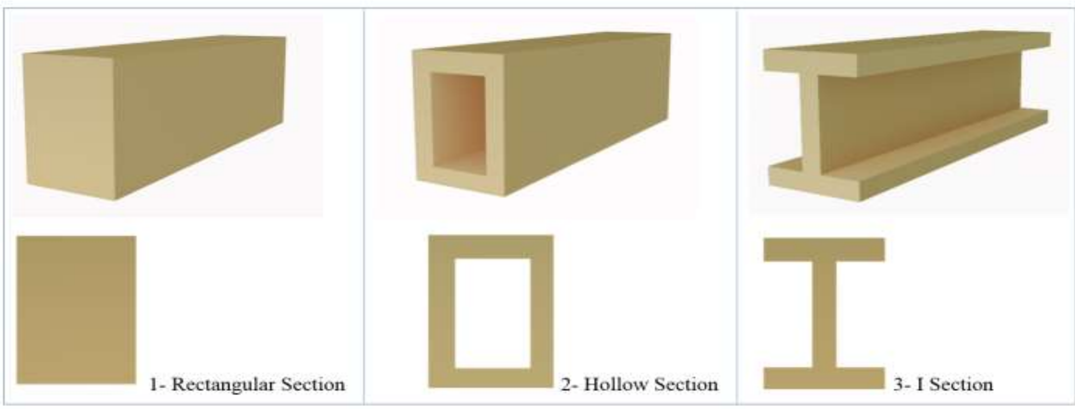


Fig. 3. Cross-Section profiles to be examined

On one hand the simplicity of their easy fabrication is to simplify computation based on symmetrical section; on the other hand, the neutral axis of the cross section is always located in the center of the beam leading to a lot of ease in calculation.

Calculations use the results of Rectangular-Section as the original, then the results of Hollow-Section and I-Section are taken into the comparison. Principles of comparisons rely on the resizing of the entire Hollow Section and I Section beam cross section provided that the area of the material is unchanged. To explore the applicability of numerical method for these categories, the dimensions of these factors are set up.

Shape Factor Study

Cross-Section Profiles

Rectangular-Section

The Rectangular Section is considered as the original shape with the total area S, contributed by 2 dimensions width *b* and height *h* in the condition b is always less than h ($b < h$)

Hollow-Section

Hollow Section is determined by offset methods from the original rectangular while the area S of materials is unchanged, in which the hollow section is ascending. The hollow area is set from the basic value S/2 reaching to S and then 2S but the material area still remains at S value. By determining the hollow area as a half, as equal and double the material area respectively, the new dimension of each turn can be calculated and shown in *Table 1*. The three Hollow sections after being modified and calculated are described as *Figure 5*. The Hollow sections (*H4… Hn*) may be calculated by this method as well, but they are not described in this image. The results, however, are shown in the graphs.

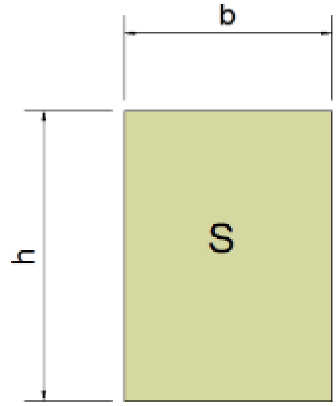
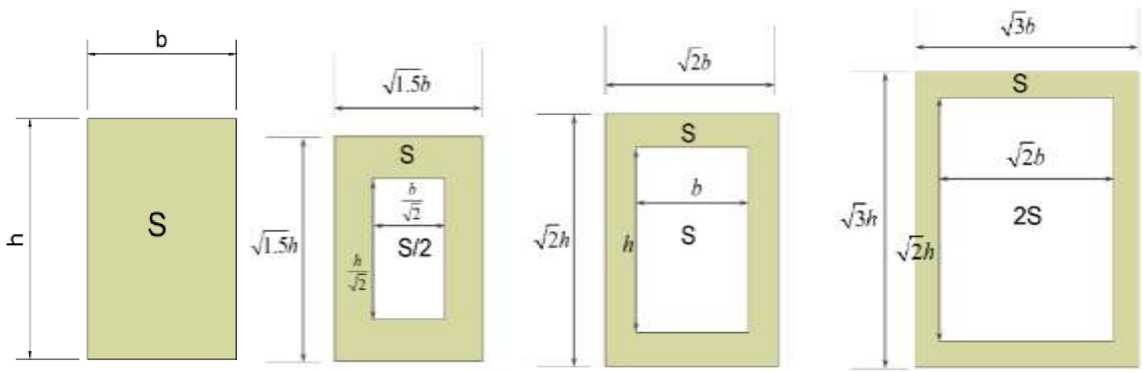
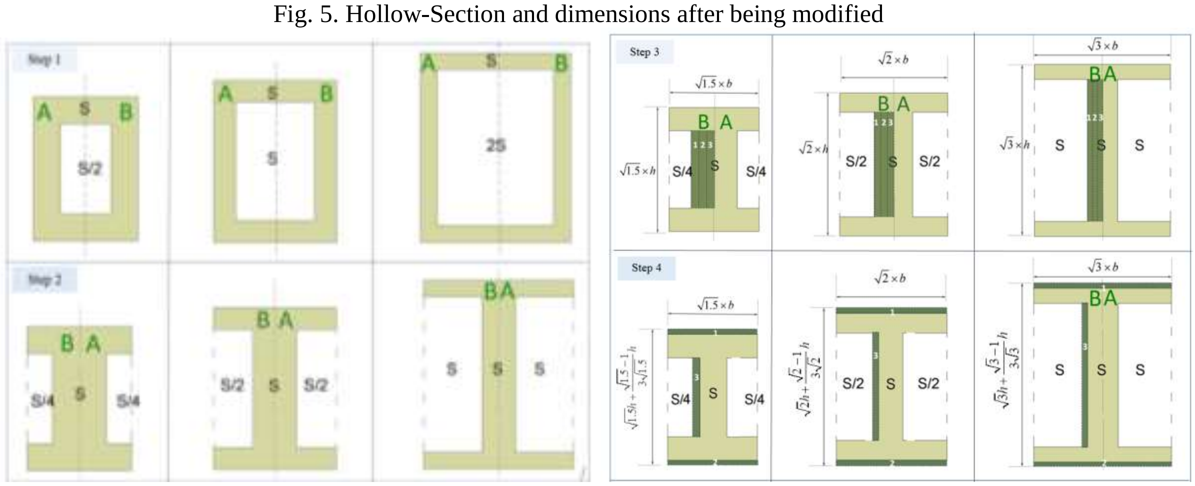


Fig. 4. Rectangular Section



Hollow section Hollow sect Hollow section (*H3)*

Fig. 5. Hollow-Section and dimensions after being modified



I1 I2 I3 I1 I2 I3

Fig. 6. I-Section and dimensions after being modified by 2 steps

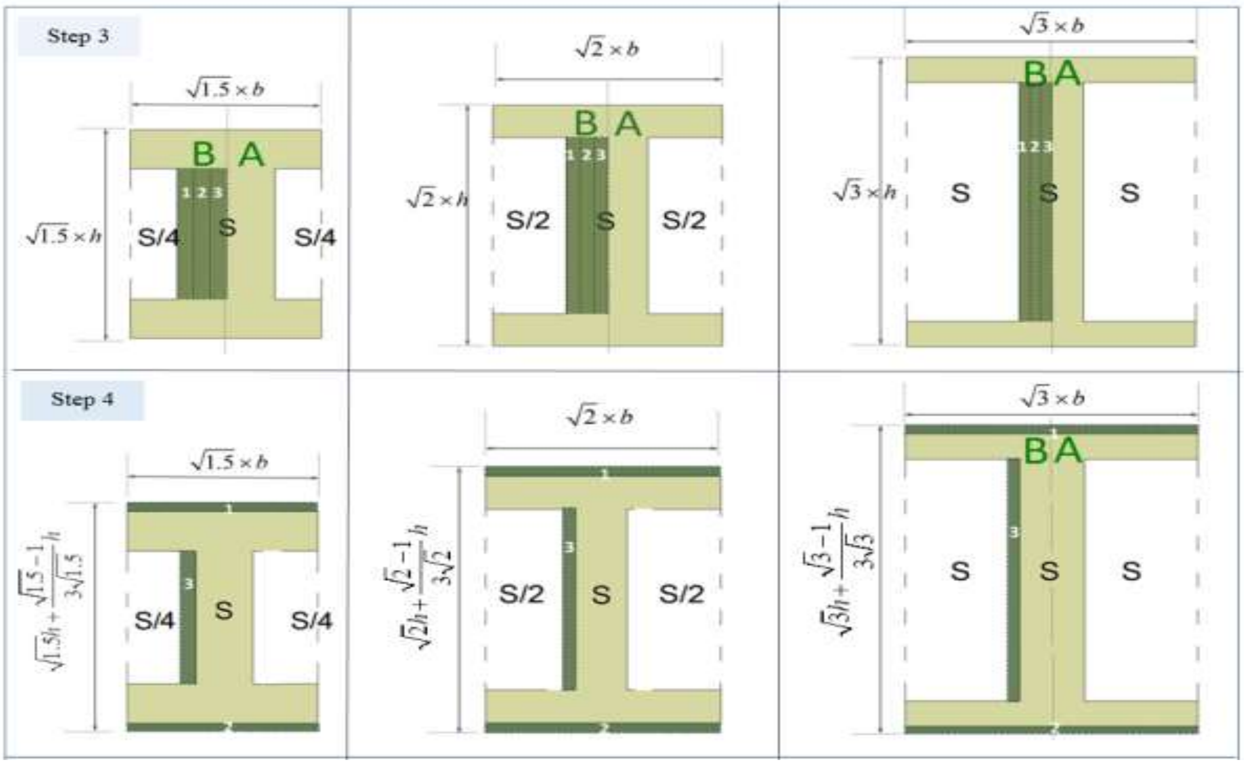


Fig. 7. I-Section and dimensions after being modified by 4 steps

I-Section

Similar to the way that Hollow-Sections are determined, I-Sections are also specified. Firstly, Hollow-Sections are separated into 2 parts (Step 1) then they are reassembled. In the other word, the I-Sections are determined by using symmetry version of Hollow Sections. I-Sections (*I4… In*) may be calculated by this method as well (*Figure 6*) and the results are shown in the graphs.

In Step 2, it is noticed that the web plate thickness of I-Sections is equal to 2 times that value of each side of Hollow-section beams. And this thickness is easily found as ineffective way of web plate. Therefore, half the area of the web plate I-Sections was taken and divided into 3 parts (Step 3) and was finally separated on the whole I shape in order to modify the thickness of the I-Section almost evenly in the flange and web plate of the beam (Step 4). Then, the new dimension of I-Section version was recalculated.

Application Methods of Calculation

Based on the relevant formula (E1) with the *b* x *h* dimensions, the value of *Moment of inertia (I)* is calculated. It is assumed that α is the ratio of the *Moment of inertia* after modifying *(Ii)* and the initial rectangular value *(I)*, $I_i = \alpha_i I = \alpha_i \times \frac{bh^3}{12}$ so $\alpha_i = I_i / I$, this α_i value is calculated and shown in Table 1.

While the value of *Section modulus (S)* is defined based on (E2), it is assumed β is the ratio of the *Section modulus* after modifying *(Si)* and the initial rectangular value *(S)*, $\beta = S_i / S$, This β value is calculated and shown in Table 2.

The equation (E3) and (E4) shows the Maximum bending stress at mid span (σ) and maximum deflection at mid span, then the ratio between modified Cross-Section and initial Rectangular-section ($\frac{l_i}{l}$) is identified, thus, the ratio (E5) may give the idea how many times of the length span the modified beam can reach over the original beam, this value is calculated and shown in Table 3.

Second Shape Factor Study

Cross-Section Profiles

Hollow-Section

In Splitting geometric method Hollow-Section is determined by dividing the original rectangular into 4 parts, *Figure 8*. The two parts A and B will be arranged into two sides of the hollow section while the other parts C and D are arranged horizontally and placed on the bottom edges of the beam. In that way the first morphology of Hollow-Sections is specified *(H1)*, the second morphology of hollow sections (*H2*) is determined by rotating (*H1*). However, it is clearly found that the second morphology (*H2*) has a lower efficiency than (*H1*) because of its height ($\frac{h}{2}+\frac{2b}{3}$)<h

Section

By dividing the original rectangular vertically into 3 equal parts, *Figure 9*, I-Section is easily specified. The two parts A and C are rotated and re-organized into the two flanges of beams at the top and bottom. The rest of the part B serves as web beams.

Application Methods of Calculation

To simplify the calculation and establish a clear relationship of Moment of inertia (I) between cross section patterns, it is given a value α and is determined by:

$I_i = \alpha_i I = \alpha_i \times \frac{bh^3}{12}$ (E7), where I_i is considered as Moment of inertia at the section named "*i*". At this stage α value of Rectangular-Section, I-Section (α_I), and Hollow-Section (α_{H_1} α_{H_2}), will be calculated and a synthesis will be done as below:

Rectangular-Section (original) : $I = \frac{bh^3}{12}$

- I-Section: $\alpha_I = \frac{8}{27}\left(\frac{b}{h}\right)^2 + \frac{4}{3}\left(\frac{b}{h}\right) + \frac{7}{3}$

- Hollow-Section: $\alpha_{H_1} = \frac{4}{27}\left(\frac{b}{h}\right)^2 - \frac{2}{3}\left(\frac{b}{h}\right) + \frac{5}{3}$ and

$$\alpha_{H_2} = \frac{8}{27}\left(\frac{b}{h}\right)^2 + \frac{2}{3}\left(\frac{b}{h}\right) + \frac{7}{12}$$

If we call $x = \frac{b}{h}$, the relationship between α and x (*b/h*) is considered as a quadratic equation. Because of a pure beam, it is always considered that b<h, so $x=b/h \in (0,1)$, we have a graph indicating the variation of α in Graph 1.

While the values of *Section modulus (S) are defined based on (E2)* $S = S_t = S_b = \frac{\frac{bh^3}{12}}{\frac{1}{2}h}$, it is assumed that β is the ratio of the *Section modulus* after modifying *(Si)* and the initial rectangular value *(S)*, $\frac{S_i}{S} = \beta_i \times \frac{bh^3}{12}$. At this stage, this β value of Rectangular-Section, I-Section β_I, and Hollow-Section β_{H_1} β_{H_2}, will be calculated and a synthesis will be done as follows:

$$\beta_I = \frac{\frac{8}{27}\left(\frac{b}{h}\right)^2 + \frac{4}{3}\left(\frac{b}{h}\right) + \frac{7}{3}}{1+\frac{2}{3}.\frac{b}{h}};\qquad \beta_{H1} = \frac{4}{27}\left(\frac{b}{h}\right)^2 - \frac{2}{3}\left(\frac{b}{h}\right) + \frac{5}{3} \quad \text{and}$$

$$\beta_{H2} = \frac{\frac{8}{27}\left(\frac{b}{h}\right)^2 + \frac{2}{3}\left(\frac{b}{h}\right) + \frac{7}{12}}{0.5+\frac{2}{3}.\frac{b}{h}}$$

If we also call $x = \frac{b}{h}$, the relationship between β and x (*b/h*) is determined as an equation. Because of a pure beam, it is always considered that b<h, so $x=b/h$ is always between the value (0,1), we have a table and graph indicating the variation of β in Graph 2.

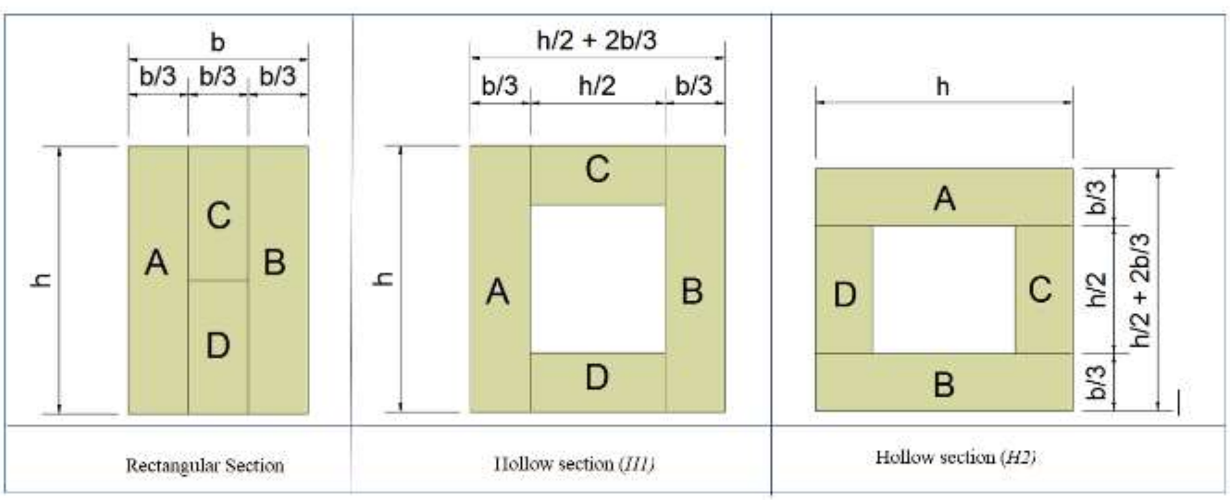


Fig. 8. Hollow-Section after being modified by splitting geometric method

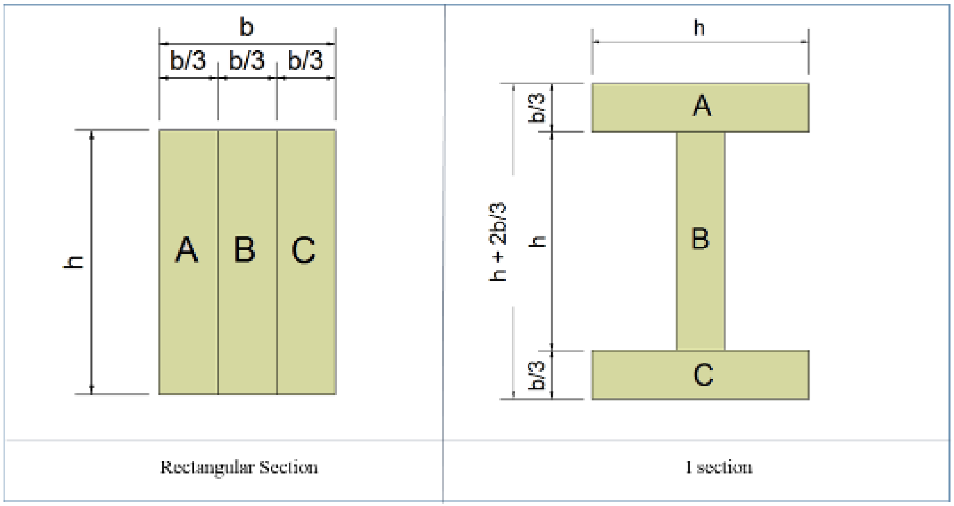


Fig. 9. I-Section after being modified by splitting geometric method

DATA ANALYSIS

The First Result of Shape Factor Efficiency

TABLE 1

THE RATIO BETWEEN MODIFIED CROSS-SECTION, MOMENT OF INERTIA VALUES AND BASIC MOMENT OF INERTIA $\alpha = I_i / I$

Section type	Rectangular	Hollow section		I'-section	
$\alpha = I_i / I$	1	*H1*	1.25	*I'1*	1.53
		H2	3	*I'2*	3.75
		H3	8	*I'3*	10.14
		H4	11.25	*I'4*	14.27

TABLE 2

THE RATIO BETWEEN MODIFIED CROSS-SECTION, SECTION MODULUS VALUE AND BASIC SECTION MODULUS VALUE $\beta = S_i / S$

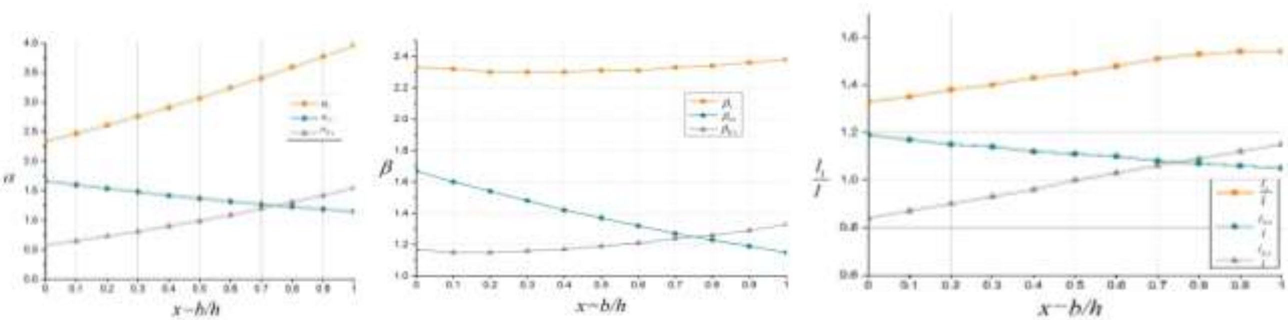
Section type	Rectangular	Hollow section		I'-section	
$\beta = S_i / S$	1	*H1*	2.04	*I'1*	2.5
		H2	4.24	*I'2*	5.3
		H3	9.24	*I'3*	11.7
		H4	12.01	*I'4*	15.26

TABLE 3
THE RATIO OF THE LENGTH SPAN L BETWEEN MODIFIED CROSS-SECTION AND REC-SECTION

	$\frac{l_{Hi}}{l_o}=\min\left(\sqrt{\frac{S_{Hi}}{S_o}},\sqrt[3]{\frac{I_{Hi}}{I_o}}\right)$		$\frac{l_I}{l}=\min\left(\sqrt{\frac{S_{Ii}}{S_o}},\sqrt[3]{\frac{I_{Ii}}{I_o}}\right)$
H1	1.1	*I'1*	1.2
H2	1.4	*I'2*	1.6
H3	2.0	*I'3*	2.2
H4	2.2	*I'4*	2.4

The results of Table 3 may notice that I-Section demonstrated a higher efficiency length of span than the Rectangular and Hollow-Section, in terms of the same material and the same cross-sectional area of the beam. In addition, this approach also showed that the overall area of each incremental beam, apparently, decreased their efficiency, i.e. *H4* (2.78) compared with *H1* (3,91) means that the larger Hollow-Section version has a lower performance than the smaller version. This occurred analogous to the I-Section as well.

The Result of Second Shape Factor Study



(a) (b) (c)

Graph 1. (a)The ratio between modified Cross-Sectional Moment of inertia α
(b)The ratio between modified Cross-Sectional Section modulus β
(c)The length span ratio between modified Cross-Sections $\frac{l_i}{l}$ and Rec-Section

TABLE 5
THE COMPARISON LI VALUE AND BASIC L VALUE BY NUMBER

$x=\frac{b}{h}$	$\frac{l_I}{l_o}=\min\left(\sqrt{\frac{S_I}{S_o}},\sqrt[3]{\frac{I_I}{I_o}}\right)$	$\frac{l_{H1}}{l_o}=\min\left(\sqrt{\frac{S_{H1}}{S_o}},\sqrt[3]{\frac{I_{H1}}{I_o}}\right)$	$\frac{l_{H2}}{l_o}=\min\left(\sqrt{\frac{S_{H2}}{S_o}},\sqrt[3]{\frac{I_{H2}}{I_o}}\right)$
0.0	1.33	1.19	0.84
0.1	1.35	1.17	0.87
0.2	1.38	1.15	0.90
0.3	1.40	1.14	0.93
0.4	1.43	1.12	0.96
0.5	1.45	1.11	1.00
0.6	1.48	1.10	1.03
0.7	1.51	1.08	1.06
0.8	1.53	1.07	1.09
0.9	1.54	1.06	1.12
1.0	1.54	1.05	1.15

The ratio between modified Cross-Section and initial Rectangular-section ($\frac{l_i}{l}$) is identified by relying on the minimum value the ratio (E5). The equation (E6) gives the idea how many times of the length span the modified beam can reach over the original beam, this value is calculated and shown as Graph 3. From this graph, we can see that, although two types of Hollow-Section (H1,H2) sometimes show a lower or higher than value 1 of Rectangular-Section, the I-Section is always higher than the three values and demonstrated an outstanding performance in all. In other words, I-Section is the optimal section in long span compared with the rectangular and hollow-section.

DISCUSSION

From the conclusion that I-beam may always represent a more optimal efficiency [9,10], there are several methods that suggested to improve I-Section in next study, such as dividing into 2 parts instead of 3 parts on a half web plate. This morphology does show a higher performance than the morphology of Step 4:

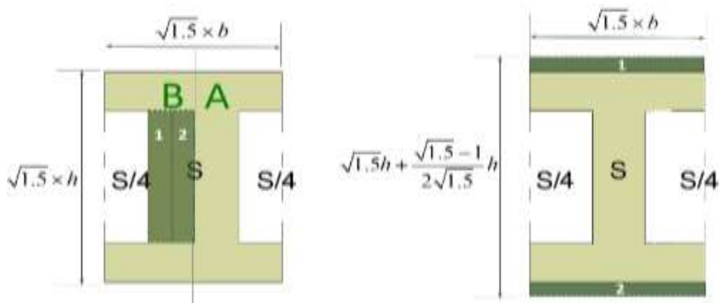


Fig. 10. Additional method I-Section

TABLE 6
THE RATIO BETWEEN MODIFIED CROSS-SECTION L VALUE AND BASIC L VALUE

$\frac{l_{I'}}{l_o} = \min\left(\sqrt{\frac{S_{I'}}{S_o}}, \sqrt[3]{\frac{I_{I'}}{I_o}}\right)$		$\frac{l_{I''}}{l_o} = \min\left(\sqrt{\frac{S_{I''}}{S_o}}, \sqrt[3]{\frac{I_{I''}}{I_o}}\right)$	
I’1	4.19	*I”1*	4.32
I’2	3.56	*I”2*	3.68
I’3	3.12	*I”3*	3.24
I’4	3.02	*I”4*	3.13

However, from Table 3 and Table 6, they could not be efficient by increasing the total area (reducing the thickness of material and enlarging hollow area), thus, it is suggested that the thickness of I-Section should be fixed and the shape factor then be conducted. By following this approach, optimizing I-Section will become more effective to identify what is the most efficient division which may be suggested in the next research.

CONCLUSION AND RECOMMEDNATIONS

From the numerical methodology of shape factor, it is found that with the same area condition and regardless of material, I-Section would indicate a higher work efficiency than the Rectangular-Section and Hollow-Section. Once the effectiveness of the I-Section would be proved, the optimization should focus on morphological study of the I-Section. The method of Shape factor that has been proposed in this study is not the most optimal method since there are many methods of splitting that rely on this approach. And the study of the optimal design shape factor applying to Flitch beam is expected with higher efficiency in the next research.

Declaration of Conflicting Interests

There are no conflicts of interest in this study.

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