Numerical Study on Optimization of Wooden-Steel Hybrid Beams Based on Shape Factor of Steel Component

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NUMERICAL STUDY ON OPTIMIZATION OF WOODEN-STEEL HYBRID BEAMS BASED ON SHAPE FACTOR OF STEEL COMPONENT

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Efficient Hybrid Steel-Timber Structural System Flitch-Beams Cross-Sections Shape Factor Optimized Morphology

INTRODUCTION
In order to pursue sustainable energy, conserving energy and reducing CO₂ emissions should be the leading tasks. Most of CO₂ emission in the world is produced from the Building Construction, there is 47% CO₂ emission coming from Buildings while the figures for Transportation and Industry are 33% and 19% respectively [1]. Concrete production contributes 5 per cent to annual anthropogenic global CO₂ production [2]. A recent study [3] shows that each 136 m² floor of Wooden construction discharges 8,331kg of CO₂, and building with a Wooden structure can give off a fixed amount of CO₂ i.e. 26,524kg (equivalent sequestration of 17,227kg). For those reasons, steel and timber hybridization are less common but they still potentially exist.

Despite of the incompatibility between the different material properties, the light, cheap, and environment-friendly nature of wood makes it a good material to pair with something strong and ductile [4]. In this research topic several different cross-sectional alternatives were given and compared. The three different types of factors are Rectangular-section, Hollow-section and I-section which are primarily shown by Moment of inertia (I) between different sections. The numerical study is to analyze stress in different cross-sections because Elastic theory is mainly determined by cross-sectional beams. The acquisition of factors was defined and thus optimal cross section morphology was determined.

In the future research, the optimal cross section can be applied into the hybridization called “Flitch- beams” to develop a method that provides a searchable manual, in which the higher efficiency of the single timber-steel-hybrid beams is suggested and can help engineers solve the sub-problem of optimizing single beams.

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LITERATURE REVIEW

Buildings in Japan have been constructed by using timber since olden times. The five-storey Kanazawa M building was constructed in 2004 in Japan [5]. The columns, beams and braces of the building consist of timber with built-in steel materials. The cross section of each member is shown in Fig. 1 which reports the structural and fire resistance capability.

The idea is to optimize the cross section of the timber-steel hybrid beam and the steel is to be placed inside the wood (Fig. 2), in which fire is prevented and both materials, under a common deformation, can reach at the same time its performance limit value [6].

There is also a design optimum cross-section for such load-carrying members. That involves using a multi-objective evolutionary algorithm for simultaneously maximizing moment of inertias and minimizing the cross-sectional areas that has been carried out by [8], [7]. Another study choosing several different cross-section alternatives with comparison via a mathematical nonlinear optimization model is performed by [7]. After the optimization is done and optimal measurements for different cross-sections are found, the results are verified by using finite element method [8].

Fig. 1. Cross sections of column, beam, and brace of Kanazawa M building.

Fig. 2. Cross section of hybrid beams tested, Vienna, Austria.

RESEARCH MODEL

Numerical Methodology of Shape Factor
Relevant Formula and Definitions Review
Moment of Inertia (I)

The second moment of area, also known as moment of inertia of plane area is a geometrical property of an area which reflects how its points are distributed with regards to an arbitrary axis. The second moment of area is typically denoted with either an $I_x$ for an axis that lies in the plane or an $I_y$ for an axis perpendicular to the plane. Its unit of dimension is length to fourth power, $L^4$. Moment of inertia value is calculated by the formula: $I_x = \frac{bh^3}{12}$ (E1).

Section Modulus (S)

Section modulus is a geometric property for a given cross-section which is used in the design of beams or flexural members. Other geometric properties used in design include area for tension, radius of gyration for compression, and moment of
inertia for stiffness. Any relationship between these properties highly depends on the shape in question. Equations for the section moduli of common shapes are given as follows:

\[ s = s_i = \frac{bh^3}{12h} \]  \hspace{0.5cm} (E2)

Maximum Bending Stress at Mid Span (\( \sigma \)) and Maximum Deflection at Mid Span (\( \Delta \))

In the field of structural engineering, the second moment of area of the cross-section of a beam is an important property which is used in the calculation of the beam’s deflection and the calculation of stress caused by a moment applied to the beam.

- The maximum bending stress at mid span:
  \[ \sigma = \frac{M}{S} \leq f_{ls} \] \hspace{1cm} (E3) where \( f_{ls} \) is allowable bending stress of timber

- The maximum deflection at mid span:
  \[ \Delta = \frac{5}{384} \times \frac{qL^4}{EI} \leq \left[ \Delta \right] \] \hspace{1cm} (E4) where \( \left[ \Delta \right] \) is allowable unit deflection

From (E3), (E4) the allowable span of section with moment M is identified by 2 compulsory conditions given characterized by two quantities S (E2) and I (E1) value, the maximum length of the beams is determined by the combination of (E*3) & (E*4) as the following formula:

\[ l \leq \min \left( \frac{f_{ls} \times 8S}{q}, \frac{384 \times \Delta/l}{EI} \right) \] \hspace{1cm} (E5)

If we have 2 sections \((I_1, S_1)(I_2, S_2)\), the ratio between maximum span \(l_1, l_2\) is:

\[ \frac{l_1}{l_2} \leq \min \left( \frac{E_1}{S_1}, \frac{E_2}{S_2} \right) \] \hspace{1cm} (E6).

This equation gives the idea of how many times of the length span the modified beam can reach over the original beam.

Shape Factors

Three different cross-section shapes are chosen for calculation (Figure 3) – including: original form: Rectangular-Section, simple form: Hollow-Section and common shape: I-Section. It is found that there is incremental complexity since the dimensions of the beam are modified at the width (b) and height (h), therefore, these shapes were chosen.

![Fig. 3. Cross-Section profiles to be examined](image)

On one hand the simplicity of their easy fabrication is to simplify computation based on symmetrical section; on the other hand, the neutral axis of the cross section is always located in the center of the beam leading to a lot of ease in calculation.

Calculations use the results of Rectangular-Section as the original, then the results of Hollow-Section and I-Section are taken into the comparison. Principles of comparisons rely on the resizing of the entire Hollow Section and I Section beam cross section provided that the area of the material is unchanged. To explore the applicability of numerical method for these categories, the dimensions of these factors are set up.
the material area still remains at S value. By determining the hollow area as a half, as equal and double the material area respectively, the new dimension of each turn can be calculated and shown in Table 1. The three Hollow sections after being modified and calculated are described as Figure 5. The Hollow sections (H4…Hn) may be calculated by this method as well, but they are not described in this image. The results, however, are shown in the graphs.

Fig. 4. Rectangular Section

Fig. 5. Hollow-Section and dimensions after being modified

Fig. 6. I-Section and dimensions after being modified by 2 steps
Application Methods of Calculation

Based on the relevant formula (E1) with the $b \times h$ dimensions, the value of Moment of Inertia ($I_l$) is calculated. It is assumed that $\alpha$ is the ratio of the Moment of Inertia after modifying ($I_i$) and the initial rectangular value ($I$),

$$I_l = \alpha I = \alpha \left( \frac{bh^3}{12} \right)$$

so $\alpha = I_i / I$, this $\alpha_i$ value is calculated and shown in Table 1.

While the value of Section modulus ($S$) is defined based on (E2), it is assumed $\beta$ is the ratio of the Section modulus after modifying ($S_i$) and the initial rectangular value ($S$), $\beta = S_i / S$.

This $\beta$ value is calculated and shown in Table 2.

The equation (E3) and (E4) shows the Maximum bending stress at mid span ($\sigma$) and maximum deflection at mid span, then the ratio between modified Cross-Section and initial Rectangular-section ($\frac{l_i}{l}$) is identified, thus, the ratio (E5) may give the idea how many times of the length span the modified beam can reach over the original beam, this value is calculated and shown in Table 3.
morphology (H2) has a lower efficiency than (H1) because of its height \( \frac{h}{2} + \frac{2b}{3} < h \)

**Section**

By dividing the original rectangular vertically into 3 equal parts, Figure 9, I-Section is easily specified. The two parts A and C are rotated and re-organized into the two flanges of beams at the top and bottom. The rest of the part B serves as web beams.

**Application Methods of Calculation**

To simplify the calculation and establish a clear relationship of Moment of inertia (I) between cross section patterns, it is given a value \( \alpha \) and is determined by:

\[
I_i = \alpha I = \alpha \times \frac{bh^3}{12} \quad \text{(E7)},
\]

where \( I_i \) is considered as Moment of inertia at the section named “i”. At this stage \( \alpha \) value of

Rectangular-Section, I-Section (\( \alpha_i \)), and Hollow-Section (\( \alpha_{H1} \), \( \alpha_{H2} \)), will be calculated and a synthesis will be done as below:

Rectangular-Section (original) : \( I = \frac{bh^3}{12} \)

- I-Section:
  \[
  \alpha_i = \frac{8}{27} \left( \frac{b}{h} \right)^2 + \frac{4}{3} \left( \frac{b}{h} \right) + \frac{7}{3}
  \]

- Hollow-Section:
  \[
  \alpha_{H1} = \frac{4}{27} \left( \frac{b}{h} \right)^2 - \frac{2}{3} \left( \frac{b}{h} \right) + \frac{5}{3}
  \]

and

\[
\alpha_{H2} = \frac{8}{27} \left( \frac{b}{h} \right)^2 + \frac{2}{3} \left( \frac{b}{h} \right) + \frac{7}{12}
\]

If we call \( x = \frac{b}{h} \), the relationship between \( \alpha \) and \( x \)

\( (b/h) \) is considered as a quadratic equation. Because of a pure beam, it is always considered that \( b < h \), so \( x = b/h \in (0,1) \), we have a graph indicating the variation of \( \alpha \) in Graph 1.

While the values of Section modulus (S) are defined based on \( (E2) S = S_i = S_b = \frac{bh^3}{2h} \), it is assumed that \( \beta \) is the ratio of the Section modulus after modifying \( (Si) \) and the initial rectangular value \( (S) \), \( \frac{S_i}{S} = \beta_i \times \frac{bh^3}{12} \). At this stage, this \( \beta \) value of

Rectangular-Section, I-Section \( \beta_i \), and Hollow-Section \( \beta_{H1}, \beta_{H2} \), will be calculated and a synthesis will be done as follows:

\[
\beta_i = \frac{8 \left( \frac{b}{h} \right)^2 + 4 \left( \frac{b}{h} \right) + \frac{7}{3}}{1 + \frac{2b}{3h}}\quad \beta_{H1} = \frac{4 \left( \frac{b}{h} \right)^2 - \frac{2}{3} \left( \frac{b}{h} \right) + \frac{5}{3}}{1 + \frac{2b}{3h}} \quad \text{and} \quad \beta_{H2} = \frac{8 \left( \frac{b}{h} \right)^2 + \frac{2}{3} \left( \frac{b}{h} \right) + \frac{7}{12}}{0.5 + \frac{2b}{3h}}
\]

If we also call \( x = \frac{b}{h} \), the relationship between \( \beta \) and \( x \)

\( (b/h) \) is determined as an equation. Because of a pure beam, it is always considered that \( b < h \), so \( x = b/h \) is always between the value \( (0,1) \), we have a table and graph indicating the variation of \( \beta \) in Graph 2.

<table>
<thead>
<tr>
<th>Rectangular Section</th>
<th>Hollow Section (1I)</th>
<th>Hollow section (H2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>h/2 + 2b/3</td>
<td>b/3</td>
<td>b/3</td>
</tr>
<tr>
<td>A</td>
<td>C</td>
<td>A</td>
</tr>
<tr>
<td>b</td>
<td>b</td>
<td>b</td>
</tr>
<tr>
<td>b/3</td>
<td>b/3</td>
<td>b/3</td>
</tr>
<tr>
<td>C</td>
<td>D</td>
<td>D</td>
</tr>
</tbody>
</table>

Fig. 8. Hollow-Section after being modified by splitting geometric method
DATA ANALYSIS
The First Result of Shape Factor Efficiency

TABLE 1
THE RATIO BETWEEN MODIFIED CROSS-SECTION, MOMENT OF INERTIA VALUES AND BASIC MOMENT OF INERTIA $\alpha = I_i / I$

<table>
<thead>
<tr>
<th>Section type</th>
<th>Rectangular</th>
<th>Hollow section</th>
<th>I'-section</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H1$</td>
<td>1.25</td>
<td>$I'1$</td>
<td>1.53</td>
</tr>
<tr>
<td>$H2$</td>
<td>3</td>
<td>$I'2$</td>
<td>3.75</td>
</tr>
<tr>
<td>$H3$</td>
<td>8</td>
<td>$I'3$</td>
<td>10.14</td>
</tr>
<tr>
<td>$H4$</td>
<td>11.25</td>
<td>$I'4$</td>
<td>14.27</td>
</tr>
</tbody>
</table>

TABLE 2
THE RATIO BETWEEN MODIFIED CROSS-SECTION, SECTION MODULUS VALUE AND BASIC SECTION MODULUS VALUE $\beta = S_i / S$

<table>
<thead>
<tr>
<th>Section type</th>
<th>Rectangular</th>
<th>Hollow section</th>
<th>I'-section</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H1$</td>
<td>2.04</td>
<td>$I'1$</td>
<td>2.5</td>
</tr>
<tr>
<td>$H2$</td>
<td>4.24</td>
<td>$I'2$</td>
<td>5.3</td>
</tr>
<tr>
<td>$H3$</td>
<td>9.24</td>
<td>$I'3$</td>
<td>11.7</td>
</tr>
<tr>
<td>$H4$</td>
<td>12.01</td>
<td>$I'4$</td>
<td>15.26</td>
</tr>
</tbody>
</table>
TABLE 3
THE RATIO OF THE LENGTH SPAN L BETWEEN MODIFIED CROSS-SECTION AND REC-SECTION

<table>
<thead>
<tr>
<th></th>
<th>( \frac{I_l}{I_o} ) (min)</th>
<th>( \frac{I_l}{I_o} ) (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1</td>
<td>1.1</td>
<td>1.1</td>
</tr>
<tr>
<td>H2</td>
<td>1.4</td>
<td>1.0</td>
</tr>
<tr>
<td>H3</td>
<td>2.0</td>
<td>1.6</td>
</tr>
<tr>
<td>H4</td>
<td>2.2</td>
<td>2.2</td>
</tr>
</tbody>
</table>

The results of Table 3 may notice that I-Section demonstrated a higher efficiency length of span than the Rectangular and Hollow-Section, in terms of the same material and the same cross-sectional area of the beam. In addition, this approach also showed that the overall area of each incremental beam, apparently, decreased their efficiency, i.e. H4 (2.78) compared with H1 (3.91) means that the larger Hollow-Section version has a lower performance than the smaller version. This occurred analogous to the I-Section as well.

The Result of Second Shape Factor Study

Graph 1. (a) The ratio between modified Cross-Sectional Moment of inertia \( \alpha \)
(b) The ratio between modified Cross-Sectional Section modulus \( \beta \)
(c) The length span ratio between modified Cross-Sections \( \frac{I}{l} \) and Rec-Section

TABLE 5
THE COMPARISON LI VALUE AND BASIC L VALUE BY NUMBER

<table>
<thead>
<tr>
<th>( \frac{x}{h} )</th>
<th>( \frac{I_l}{I_o} ) (min)</th>
<th>( \frac{I_{l1}}{I_{o1}} ) (min)</th>
<th>( \frac{I_{l2}}{I_{o2}} ) (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>1.33</td>
<td>1.19</td>
<td>0.84</td>
</tr>
<tr>
<td>0.1</td>
<td>1.35</td>
<td>1.17</td>
<td>0.87</td>
</tr>
<tr>
<td>0.2</td>
<td>1.38</td>
<td>1.15</td>
<td>0.90</td>
</tr>
<tr>
<td>0.3</td>
<td>1.40</td>
<td>1.14</td>
<td>0.93</td>
</tr>
<tr>
<td>0.4</td>
<td>1.43</td>
<td>1.12</td>
<td>0.96</td>
</tr>
<tr>
<td>0.5</td>
<td>1.45</td>
<td>1.11</td>
<td>1.00</td>
</tr>
<tr>
<td>0.6</td>
<td>1.48</td>
<td>1.10</td>
<td>1.03</td>
</tr>
<tr>
<td>0.7</td>
<td>1.51</td>
<td>1.08</td>
<td>1.06</td>
</tr>
<tr>
<td>0.8</td>
<td>1.53</td>
<td>1.07</td>
<td>1.09</td>
</tr>
<tr>
<td>0.9</td>
<td>1.54</td>
<td>1.06</td>
<td>1.12</td>
</tr>
<tr>
<td>1.0</td>
<td>1.54</td>
<td>1.05</td>
<td>1.15</td>
</tr>
</tbody>
</table>
The ratio between modified Cross-Section and initial Rectangular-section ($\frac{L'}{L''}$) is identified by relying on the minimum value the ratio (E5). The equation (E6) gives the idea how many times of the length span the modified beam can reach over the original beam, this value is calculated and shown as Graph 3. From this graph, we can see that, although two types of Hollow-Section (H1, H2) sometimes show a lower or higher than value 1 of Rectangular-Section, the I-Section is always higher than the three values and demonstrated an outstanding performance in all. In other words, I-Section is the optimal section in long span compared with the rectangular and hollow-section.

**DISCUSSION**

From the conclusion that I-beam may always represent a more optimal efficiency, there are several methods that suggested to improve I-Section in next study, such as dividing into 2 parts instead of 3 parts on a half web plate. This morphology does show a higher performance than the morphology of Step 4:

![Additional method I-Section](image)

**Fig. 10. Additional method I-Section**

**TABLE 6**

<table>
<thead>
<tr>
<th>$I'_1$</th>
<th>$I''_1$</th>
<th>$I'_2$</th>
<th>$I''_2$</th>
<th>$I'_3$</th>
<th>$I''_3$</th>
<th>$I'_4$</th>
<th>$I''_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.19</td>
<td>4.32</td>
<td>3.56</td>
<td>3.68</td>
<td>3.12</td>
<td>3.24</td>
<td>3.02</td>
<td>3.13</td>
</tr>
</tbody>
</table>

However, from Table 3 and Table 6, they could not be efficient by increasing the total area (reducing the thickness of material and enlarging hollow area), thus, it is suggested that the thickness of I-Section should be fixed and the shape factor then be conducted. By following this approach, optimizing I-Section will become more effective to identify what is the most efficient division which may be suggested in the next research.

**CONCLUSION**

From the numerical methodology of shape factor, it is found that with the same area condition and regardless of material, I-Section would indicate a higher work efficiency than the Rectangular-Section and Hollow-Section. Once the effectiveness of the I-Section would be proved, the optimization should focus on morphological study of the I-Section. The method of Shape factor that has been proposed in this study is not the most optimal method since there are many methods of splitting that rely on this approach. And the study of the optimal design shape factor applying to Flitch beam is expected with higher efficiency in the next research.

**REFERENCES**

[1] “Michael Green TED speaker,” [https://www.youtube.com/watch?v=Xi_PD5aZT7Q](https://www.youtube.com/watch?v=Xi_PD5aZT7Q)


— This article does not have any appendix. —